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Improving Grade Ten Students' Achievement in Solid Geometry through Guided Inquiry-Based Instruction Using Variation Theory

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Abstract: This study investigates the effectiveness of Guided Inquiry-Based Instruction (GIBI) integrated with Variation Theory in improving grade ten students' solid geometry achievement in Debre Tabor City, Ethiopia. A quasi-experimental design involving 99 students found in three classes from three government schools assigned them randomly to three groups: Experimental Group 1 (EG1, n=30) received GIBI with Variation Theory, Experimental Group 2 (EG2, n=37) received only GIBI and the Control Group (CG, n=32) followed traditional methods. Pre- and post-tests analyzed using ANCOVA and paired t-tests revealed significant improvements, with EG1 achieving the highest scores (p = .000). Effect sizes were substantial for EG1 (Cohen's d = 1.50) and EG2 (d = 1.39) compared to CG (d = .73). The results highlight that GIBI combined with Variation Theory significantly enhances students' solid geometry achievement, emphasizing the value of such kind of innovative teaching strategy to foster students' achievement in similar educational contexts.

Keywords: *Guided inquiry-based instruction, mathematics achievement, secondary education, solid geometry, variation theory.*

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Introduction

In the evolving landscape of the 21st century, education systems face increasing pressure to address global challenges such as climate change, economic disruption, and societal shifts (Care et al., 2018). These challenges emphasize equipping students with skills such as problem-solving, collaboration, and critical thinking. In this context, mathematics education is pivotal in developing such competencies (Adem et al., 2020; Eurydice, 2022). Despite its significance, mathematics achievement has been alarmingly low, particularly in geometry, in many developing countries, including Ethiopia, due to the traditional teaching methods (TTM) that dominate their schools (Sichangi et al., 2024).

TTM involves the teacher as the director of learning and is mainly accomplished through lectures, repetitive practice of basic skills, and constructive feedback. It is also known as direct instruction, explicit instruction, or conventional teaching (Stephan, 2020). This approach has been linked to students' struggles with abstract topics like solid geometry, which requires higher-order thinking and spatial reasoning (Demssie & Yimam, 2019; Mekuria & Teketel, 2019; Walde, 2019). To address this issue, innovative instructional strategies, such as Guided Inquiry-Based Instruction (GIBI) and variation theory, have emerged as promising alternatives (Lo, 2012; Nisa & Astriani, 2022).

GIBI, rooted in constructivist learning theory, fosters students' engagement through exploration and inquiry while allowing teachers to guide the learning process (Dorier & Maass, 2020). It encourages students to build on past experiences and use their imagination and creativity to discover facts and relationships by providing them with learning environments focusing on opportunities for inventing solutions to problems (Jumantini et al., 2021; Seel et al., 2017; Yanakit & Kaewsaiha, 2021). Studies showed the positive effect of GIBI on students' geometric achievement (Khasawneh et al., 2023; Odupe & Opeisa, 2019; Sichangi et al., 2024).

Moreover, Marton's variation learning theory and Gu's teaching through variation enhance students' understanding by

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exposing them to varied representations of concepts using different patterns of variation such as separation (i.e., contrast and generalization), fusion, conceptual, and procedural variations while controlling the others (Gu et al., 2017; Kullberg et al., 2024). These theories enable students to develop deeper abstraction and enhance higher-order thinking, and academic performance (Baskoro, 2021; Handy, 2021; Jacques, 2018). Additionally, Voon et al. (2020) recommended that integrating variation theory with constructivist approaches like GIBI improves students' mathematical achievement.

Statement of the Problem

Solid geometry, a fundamental component of secondary schools' mathematical education, is one of the most challenging areas for Ethiopian students due to its abstract nature. Their achievement has been low since 2013 according to the National Educational Assessment and Examinations Agency (NEAEA, 2017). The prevalent use of TTM in classrooms has been identified as a significant factor contributing to this decline (Aziz & Kang, 2021; Farooqi, 2020; Istikomah et al., 2022; Mekuria & Teketel, 2019). Despite evidence supporting the effectiveness of variation theory and GIBI in enhancing students' mathematics achievement, their integration remains limited globally, particularly in the Ethiopian educational system. The study addresses this literature gap by investigating whether combining these instructional approaches can significantly improve student solid geometry achievement.

Literature Review

GIBI and Mathematics Achievement

GIBI is a student-centered teaching approach rooted in constructivist theory. It empowers students to actively engage in problem-solving, experimentation, and critical thinking, with teachers providing guidance as needed (Dorier & Maass, 2020; Jumantini et al., 2021) while students assume responsibility for the inquiry process (Berhanu & Sheferaw, 2022).

Numerous studies have demonstrated GIBI's superiority over TTM in improving academic outcomes across various disciplines. For example, Asante-Mensa et al. (2024) found that GIBI significantly enhanced students' understanding of geometric concepts compared to TTM, with large effect sizes (Cohen's d = 1.13). Similarly, Khasawneh et al. (2023) reported that inquiry-based teaching enhanced student performance in mathematics, particularly in algebra. Ogunjimi and Gbadeyanka (2023) also found a strong positive effect of GIBI on students' mathematics performance. Moreover, Sichangi et al. (2024) and Odupe and Opeisa (2019) studies supported these findings, noting improved participation and higher mathematics scores among students engaged in GIBI compared to students taught by TTM.

However, some studies highlight limitations in GIBI's application. For instance, Aguguam and Okoro (2020) noted that in economics, individualized teaching strategies outperformed GIBI in certain contexts. Similarly, Richter et al. (2022) reported better outcomes with direct instruction in biology than GIBI. This inconsistency suggests that while GIBI is generally effective, its success may depend on the subject matter that shows the need for additional research, or the inclusion of complementary strategies like variation theory.

Variation Theory and Mathematics Achievement

Various theories have been proposed in geometry education to enhance students' problem-solving skills and geometric reasoning and improve their overall learning outcomes. Two prominent theories in this regard are Matron's variation learning theory and GU's teaching through variation.

Variation Theory, developed by Marton and Pang (2007), emphasizes learning by exposing students to critical features of a concept by varying some aspects using separation (i.e. contrast and generalization) and fusion patterns of variation, while keeping others constant (Kullberg et al., 2024). For example, in solid geometry, this might involve illustrating the properties of three-dimensional shapes through diagrams, nets, and physical models (Jacques, 2018). Experiencing solid shapes through different representations enables students to develop a more comprehensive understanding of their spatial characteristics and geometric attributes, which could help to improve their mathematics achievement.

Besides, teaching and learning through variation problems has been practiced in China since the 1980s. However, GU and his colleagues have theorized it and explored how to use and to increase students' achievement in mathematics and help students understand the essential features of a concept by differentiating them from non-essential features using conceptual and procedural patterns of variation (Gu et al., 2017; Lomibao & Ombay, 2017). Similarly, Lo (2012) argued that currently enrolled students need two key capabilities to solve 21st-century problems: the ability to recognize multiple problem-solving approaches (procedural variation) and to apply existing knowledge effectively (conceptual understanding) that can be fostered through instructional variation. In this study, these variation theories were used to design geometric activities.

Studies showed the positive effects of these variation theories on students' mathematics achievement by enhancing their capacity for abstraction and retention, and by facilitating conceptual understanding. For instance, Jing et al. (2017) found that students taught algebra using variation theory-based strategies significantly outperformed those taught through conventional methods. Similarly, Lomibao and Ombay (2017) demonstrated that repetition with variation led to higher mathematics achievement among grade ten students in China.

Additionally, studies by Baskoro (2021) and Handy (2021) confirmed that complementing Marton's variation theory with GU's teaching through variation academic performance in mathematics. Moreover, Voon et al. (2020) recommended that integrating variation theory with constructivist approaches like GIBI could improve students' mathematics learning and achievement. Despite these promising findings of GIBI and variation theory in improving students' mathematics understanding and achievement, their combined effect remains underexplored globally, particularly in Ethiopia's secondary schools.

This study aimed to address this critical literature gap by investigating the effects of GIBI using variation theory on grade ten students' solid geometry achievement in Debre Tabor City, Amhara region, Ethiopia. Its findings would offer a novel instructional framework to address systemic challenges in mathematics education and provide evidence-based strategies to improve learning outcomes in Ethiopia.

Research Questions

Two research questions were posed in this study.

1. Is there a significant difference in solid geometry achievement post-test scores for students using three different teaching methods (GIBI using variation theory, GIBI alone, and TTM) while controlling for their pre-test scores?
2. Which teaching method (GIBI using variation theory, GIBI alone, and TTM) is more effective in improving students' solid geometry achievement?

Methodology

Research Design

A quasi-experimental with non-equivalent control pre-test post-test design was employed to examine the effect of GIBI using variation theory on grade ten students' solid geometry achievement in Debre Tabor City, Amhara region, Ethiopia.

Table 1. Research Design Layout

Groups		Intervention	
EG1	Pre-test	GIBI using Variation Theory	Post-test
EG2	Pre-test	GIBI	Post-test
CG	Pre-test	TTM	Post-test

Note: EG1= Experimental Group 1; EG2=Experimental Group 2; CG=Control Group

Table 1 shows all study groups completed a pre-test to establish their baseline knowledge of solid geometry before the intervention and a post-test to measure their solid geometry achievement after the intervention. EG1 was instructed with GIBI using variation theory; EG2 was taught by GIBI alone, and the CG received TTM. The intervention was implemented in the second semester of the academic year 2023/2024 and lasted for four weeks with four periods per week.

Sample and Participants

Multi-stage sampling was used for this study. First, out of four government secondary schools in Debre Tabor City, three schools were selected purposively by taking their infrastructures, average class size, and availability into account. Then, three mathematics teachers (one from each school) were included as participants using purposive sampling based on their teaching experience, education level, and voluntary. Finally, three grade ten classes (one from the selected mathematics teacher's intact classes) were randomly chosen and assigned to the experimental and control groups. A total of 99 grade ten students with distribution; EG1 (n =30), EG2 (n=37), and CG (n=32) have participated in this study.

Data Collection Instrument

Quantitative data was collected using the Mathematics Achievement Test (MAT), which is prepared by the researcher based on the Ethiopian grade ten mathematics curriculum (Federal Democratic Republic of Ethiopia Ministry of Education, 2023), then ensured its face was validated by advisors, two secondary school mathematics teachers, and a mathematics lecturer of Begimidir College of Teacher Education. The test consisted of 36 multiple-choice items with four response options (A, B, C, and D), and assessed students' conceptual knowledge, conceptual understanding, and application of solid geometry concepts.

Finally, its reliability was checked through pilot testing. The calculated reliability coefficient using Kuder–Richardson formulas (K-R20) was .72, indicating an internal consistency of the test items (Pallant, 2016; Roni et al., 2020). The test is presented in the appendixes. Table 2 summarizes the distribution of MAT items by sub-topics and cognitive domains.

Table 2. MAT Items Distribution

No	Sub-Topics	Cognitive Domains		
		Conceptual Knowledge	**Conceptual understanding**	**Application**
1.	Revision of prisms and cylinders	1	3	3
2.	Pyramids, cones, and spheres	2	8	9
3.	Frustum of pyramids and cones, and composed solids	2	1	7
	Total number of Items	5	12	19

Implementation Procedures

A supplementary instructional manual was prepared by the researcher based on GIBI and variation theory principles using the 5E instructional model (Engage, Explore, Explain, Elaborate, and Evaluate) of Bybee (2014). The manual contained 16 structured solid geometry lessons covering 16 different sub-topics. Additionally, models and pictures of solid figures, and rules were used to aid the teaching-learning process.

Two mathematics teachers who taught the experimental groups (EG1 and EG2) attended a five-day training program to validate the manual contents and familiarize them with GIBI by focusing on how they will implement it in classroom settings. Additionally, one teacher who instructed EG1 received a day of special training on conducting geometry activities designed using variation theory.

Following these, EG1 and EG2 students were organized by their teachers into six heterogeneous groups of four to five students in a group, and then EG1 lessons focused on driving the surface area and volume formula of solid figures by incorporating activities that highlight critical features of geometric concepts (e.g., identifying similarities and differences among solid figures) whereas EG2 lessons focused on driving the surface area and volume formula without explicit use of variation theory. Meanwhile, the CG students received TTM in their natural seating arrangement.

The intervention took four weeks with four periods per week. Besides, to prevent treatment diffusion, the study group's students were unaware of the different teaching methods they were receiving. All stages of the implementation procedures were conducted with careful attention to ethical considerations such as informed consent, confidentiality, and data protection throughout the study.

Data Analysis

We performed preliminary analysis to make sure that the study group's mathematics achievement pre-test scores (MAPreTS) data met or not ANOVA assumptions. The Shapiro-Wilk and Levene's tests showed that the groups' pre-test data met ANOVA assumptions (see Table A1) in the appendixes. Additionally, the ANOVA results showed a significant difference across the study groups in their MAPreTS (F (2, 101) = 12.816, p = .000) (see Table A2) in the appendixes, indicating the inequality of the groups before the intervention. This notifies the use of analysis of covariance (ANCOVA) to compare their solid geometry achievement differences after the intervention. Similarly, the groups' mathematics achievement post-test scores (MAPostTS) data passed through ANCOVA assumptions tests, and the results showed that all groups' data met ANCOVA assumptions (i.e. normality checked through Shapiro-Wilk test; equality of error variances was ensured by Levene's test; homogeneity of regression slopes tested by executing ANCOV with interaction effect; and finally linearity between the dependent variable and the covariate was confirmed by scatter plot) as shown in Tables A3-A5, and Figure A1 in the appendixes.

For the main analysis, descriptive statistics (mean and standard deviation) were used to summarize the students' MAPostTS. Additionally, we performed ANCOVA to compare the groups' differences in their MAPostTS, while controlling their MAPreTS. Finally, a paired sample t-test was executed to examine the three study group's achievement improvement due to the intervention.

Results

Groups Difference in their MAPostTS

Descriptive statistics in Table 3 show that EG1 had a mean score of 12.27 (SD = 3.413); EG2 had a mean of 10.76 (SD = 2.326); and CG had a mean of 7.13 (SD = 1.809), indicating the difference among study groups in their MAPostTS. However, further analysis is needed to ensure these differences are statistically significant.

Table 3. Descriptive Statistics of MAPostTS

Groups	Mean	Std. Deviation	N
EG1	12.27	3.413	30
EG2	10.76	2.326	37
CG	7.13	1.809	32
Total	10.04	3.310	99

A one-way between-groups ANCOVA was calculated to examine the effect of teaching methods on students' MAPostTS while controlling the effect of MAPreTS. Based on our preliminary analyses, the study groups were significantly different in their MAPreTS (F (1, 95) = 28.68, p = .000). See Table A2 in the appendixes.

Table 4. ANCOVA Result of MAPostTS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	586.729	3	195.576	38.143	.000	.546
Intercept	936.445	1	936.445	182.633	.000	.658
MAPreTS	147.069	1	147.069	28.683	.000	.232
Groups	203.908	2	101.954	19.884	.000	.295
Error	487.109	95	5.127			
Total	11054.000	99				
Corrected Total	1073.838	98				

Table 4 shows that the main effect for teaching methods was significant (F (2, 95) = 19.88, p = .000) with EG1 who taught with GIBI using variation significantly outperformed (M = 12.27, SD = 3.41) EG2 who instructed by GIBI alone (M = 10.76, SD = 2.33), and CG who received TTM (M = 7.13, SD = 1.81). The large effect size (η^2 = .295) shows that GIBI using variation theory had a significant effect on students' solid geometry achievement.

Effectiveness of GIBI using variation theory, GIBI alone, and TTM

A paired samples t-test was performed to examine the effectiveness of the three teaching methods in improving the study groups' solid geometry achievement.

Table 5. Paired Samples t- Test Results

				95% Confidence Interval of the Difference					
		Mean	Std. Dev.	Lower	Upper	t	df	Sig.	Cohen's d
EG1	MAPostTS - MAPreTS	3.53333	2.35962	2.65224	4.41443	8.202	29	.000	1.497
EG2	MAPostTS - MAPreTS	4.40541	3.16631	3.34970	5.46111	8.463	36	.000	1.391
CG	MAPostTS - MAPreTS	2.25000	3.09005	1.13592	3.36408	4.119	31	.000	0.728

Results in Table 5 revealed a significant change in MAPostTS compared to MAPreTS for all three groups. In EG1, there was a statistically significant increase in scores (t (29) = 8.20, p = .000), with a large effect size (d = 1.50), indicating that integration variation theory with GIBI had a strong effect on MAPostTS. Similarly, EG2 showed a significant improvement (t (36) = 8.46, p = .000), also with a large effect size (d = 1.39). This suggests that GIBI alone was also effective in enhancing MAPostTS, though the effect size was slightly lower compared to EG1.

Moreover, the CG demonstrated a statistically significant, though smaller, improvement in MAPostTS scores (t (31) = 4.12, p = .000), with a medium effect size (d = .73). This finding suggests that while some improvement occurred without intervention, the magnitude of change in CG was notably less than in EG1 and EG2, emphasizing the importance of the interventions in driving greater improvements in MAPostTS scores.

Discussion

The purpose of the study was to examine the effect of GIBI using variation theory on grade ten students' solid geometry achievement post-test scores in Debre Tabor City, Amhara region, Ethiopia. Besides, the study evaluates the effectiveness of the three teaching methods in improving students' achievement in solid geometry.

The ANCOVA result revealed significant differences in MAPostTS scores across the three study groups. EG1 achieved the highest scores, followed by EG2, with CG scoring the lowest. This outcome indicates that GIBI alone and GIBI using variation theory were more effective in enhancing students' MAPostTS compared to. These findings are consistent with previous research findings that confirmed the benefits of GIBI in enhancing students' achievement when compared with TTM (Ogunjimi & Gbadeyanka, 2023; Yolida & Marpaung, 2023).

Additionally, the paired t-test results show a greater achievement improvement observed in EG1 with effect size (d = 1.50) compared to EG2 with effect size (d = 1.39), suggesting that combining variation theory with GIBI is more impactful than employing GIBI alone. This finding is in line with previous research findings that supported the value of variation theory-informed mathematics pedagogy in improving students' achievement (Baskoro, 2021; Jing et al., 2017;

Lomibao & Ombay, 2017; Voon et al., 2020). It also shows the substantial contribution of variation theory in enhancing students' solid geometry achievement by strengthening their understanding of geometric concepts through multiple representations as theorized by Marton and Gu (Gu et al., 2017; Kullberg et al., 2024), and suggested for secondary schools' mathematics abstract topics like solid geometry (Handy, 2021).

Furthermore, the underperformance of CG compared to EG1 and EG2 highlights the limited effectiveness of TTM as a teaching strategy for improving students' MAPostTS.

This finding contradicts previous research findings that reported the positive effects of TTM in enhancing students' achievement in different subjects when compared with GIBI (Aguguam & Okoro, 2020; Richter et al., 2022). However, this finding emphasizes the importance of well-structured and theoretically grounded interventions like GIBI using variation theory to drive meaningful progress in solid geometry.

These findings have important implications for geometry education. Mathematics teachers can create a learning environment that permits students to explore different solid geometry concepts through multiple representations using activities designed based on variation theory principles. They can also encourage students to be actively engaged during the inquiry process through their guidance to enhance students' solid geometry achievement.

Conclusion

From the ANCOVA and paired t-test findings of this study, we conclude that integration variation theory with GIBI is effective in improving Ethiopian secondary school students' achievement in solid geometry when compared with using GIBI alone, and TTM. The clear benefits of EG1 suggest that future research should focus on identifying the specific elements driving its success, exploring its scalability, and assessing its applicability in various educational settings and mathematics strands to ensure its sustained and widespread impact.

Recommendations

Based on the above findings, we made the following recommendations.

- The Ethiopian Ministry of Education should integrate this instructional approach into the national mathematics curriculum. This method could enhance achievement not only in geometry but also in other complex mathematics topics.
- Comprehensive training programs should be implemented to equip mathematics teachers with the skills needed to apply GIBI and Variation Theory effectively. Training should focus on designing variation-based lesson plans and facilitating inquiry-based activities in classrooms, as recommended by Gu et al. (2017).
- Developing countries whose educational context is similar to Ethiopia can adapt this teaching strategy to enhance their students' solid geometry achievement.

Limitations

The study had limitations that should be addressed in future research. First, the intervention lasted only four weeks, which may have constrained the observation of the long-term effects of GIBI using variation theory on students' solid geometry achievement, suggesting the need for extended intervention periods to capture sustained effects. Second, the sample was limited to three government schools in a single city, which restricts the generalizability of the findings; incorporating rural and private schools could provide a more comprehensive understanding of the approach's effectiveness. Third, the research focused exclusively on solid geometry, excluding other mathematical topics, highlighting the need to explore its application to areas like algebra, calculus, or statistics.

Ethics Statement

Subjects involved in this study signed an agreement providing their informed consent to participate in the study. The study was reviewed and approved by Hawassa University.

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Conflict of interest

The authors declare that they have no competing interests.

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Authorship Contribution Statement

Yeshanew: Conceptualization, design, methodology, analysis, writing, software. Belachew: Supervision, methodology, validation, editing/reviewing. Gezahegn: Supervision, methodology, editing/reviewing. Tesfa: Supervision, editing/reviewing.

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Appendixes

Mathematics Achievement Test

School Name ___________ **ID. No** ____________ **Time allotted:** 80 min.

Instruction: *Please circle the letter of your choice from the given options.*

1. Which of the following is **not** a prism?

A) Cube B) Rectangular solid C) Parallelepiped D) Tetrahedron

2. What plane figures are the lateral faces of *right regular* pyramids?

A) Equilateral triangles B) Trapeziums C) Isosceles triangles D) Rectangles

3. The lateral face of *a frustum* of a right cone is _______

A) Sector of annulus B) Trapezium C) Circle D) Isosceles triangles

4. Which of the following objects has a spherical shape?

A) Tea Cup B) Matchbox C) Moon D) Happy Birthday cap

5. The lateral faces of a *frustum* of a regular pyramid are _________

A) Rectangles B) Isosceles Trapeziums C) Trapeziums D) Parallelogram

Instruction: *Items numbered from 6 to 17 have two options. So, make sure to select two options for these items.*

6. The radius of a spherical balloon increases from 7 cm to 14 cm when air is pumped into it. The ratio of the surface area of the original balloon to the inflated one is ______

A) 1: 2 B) 2: 1 C) 1: 4 D) None of these

The reason for your answer above is

Let *r* be the radius of a spherical balloon. Then the surface area (SA) of the balloon becomes$4\pi r^2$. So,

A) $\frac{SA\ of\ the\ original\ balloon}{SA\ of\ the\ inflated\ balloon} = \frac{4\pi(7)^2}{4\pi(14)^2} = \frac{14}{28} = \frac{1}{2}$ (I.e. 1:2)

B) $\frac{SA\ of\ the\ original\ balloon}{SA\ of\ the\ inflated\ balloon} = \frac{4\pi(7)^2}{4\pi(14)^2} = (\frac{1}{2})^2 = \frac{1}{4}$ (I.e. 1: 4)

C) $\frac{SA\ of\ the\ inflated\ balloon}{SA\ of\ the\ original\ balloon} = \frac{4\pi(14)^2}{4\pi(7)^2} = \frac{28}{14} = 2$ (I.e. 2: 1)

D) None of these

7. If the diagonal of a cube is *d* cm, what is the volume of the cube? (In cm^3)

A) $\frac{d^3}{\sqrt{3}}$ B) $\frac{d^3}{\sqrt{2}}$ C) $\frac{d^3}{3\sqrt{3}}$ D) $\frac{d^3}{2\sqrt{2}}$

The reason for your answer above is

If the length of one side of a cube is ***a*** cm, then its volume (***V***) becomes a^3. Thus,

A) $a = \frac{d}{\sqrt{3}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{3}})^3 = \frac{d^3}{3\sqrt{3}}$ B) $a = \frac{d}{\sqrt{2}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{2}})^3 = \frac{d^3}{2\sqrt{2}}$

C) $a = \frac{d}{\sqrt{3}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{3}})^3 = \frac{d^3}{\sqrt{3}}$ D) $a = \frac{d}{\sqrt{2}} \Rightarrow V = a^3 = (\frac{d}{\sqrt{2}})^3 = \frac{d^3}{\sqrt{2}}$

8. If the total surface area of a cube is $x\ cm^2$ and its volume is$\frac{x\sqrt{3}}{3}\ cm^3$, find the *Main diagonal* of the cube.

A) 6 B) 4 C) $2\sqrt{6}$ D) 3

The reason for your answer above is:

Let ***a*** and ***c*** be the length of one edge and side diagonal of the cube respectively. Then

A) $4a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{4}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{4}) = \frac{x\sqrt{3}}{3} \Rightarrow a = \frac{4\sqrt{3}}{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}\left(\frac{4\sqrt{3}}{3}\right) = 4$

B) $3a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{3}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{3}) = \frac{x\sqrt{3}}{3} \Rightarrow a = \sqrt{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}(\sqrt{3}) = 3$

C) $6a^2 = x\ \&\ a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{6}$, but $c = a\sqrt{2}$ So, $a^3 = a(a^2) = a(\frac{x}{6}) = \frac{x\sqrt{3}}{3} \Rightarrow a = 2\sqrt{3} \Rightarrow d = a\sqrt{2} = \sqrt{2}(2\sqrt{3}) = 2\sqrt{6}$

D) $6a^2 = x$ & $a^3 = \frac{x\sqrt{3}}{3} \Rightarrow a^2 = \frac{x}{6}$, but $c = a\sqrt{3}$ So, $a^3 = a(a^2) = a(\frac{x}{6}) = \frac{x\sqrt{3}}{3} \Rightarrow a = 2\sqrt{3} \Rightarrow d = a\sqrt{3} = \sqrt{3}(2\sqrt{3}) = 6$

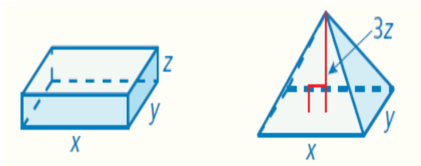
9. Which of the following is **true** about the volume of the two solids figures shown below? Why?

A) The volume of the prism is *greater than* the volume of the pyramid

B) They have the same volume

C) The volume of the prism is *less than* the volume of the pyramid

D) I can't compare their volume



The reason for your answer above is

A) Although the height of the pyramid is *three times* higher than the height of the prism, the volume of the prism is greater than the volume of the pyramid

B) The volume of the prism is the base area time height (i.e. XYZ) and the volume of the pyramid is *one-third* times the base area time's height (i.e. XYZ). So, they have the same volume.

C) I can't compare their volume because their dimensions are only variables.

D) The volume of the prism is *one-third* times the base area time height (i.e.$\frac{1}{3}$XYZ) and the volume of the pyramid is the base area time height (i.e. 3XYZ). So, the volume of the prism is less than the volume of the pyramid

10. The sum of the base area of a cylinder is **equal** to its lateral face area. If the altitude of the cylinder is 2 cm, then its volume is ______

A) $\frac{32}{3}\pi\ cm^3$ B) $4\pi\ cm^3$ C) $16\pi\ cm^3$ D) $8\pi cm^3$

The reason for your answer above is

Let ***r*** and ***h*** be the radius and height of the cylinder respectively. Then,

A) $2\pi r^2 = 2\pi rh$ and $h = 2 \Rightarrow r = 2$. So, its volume $(V) = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(2) = 4\pi$

B) $\pi r^2 = 2\pi rh$ and $h = 2 \Rightarrow r = 4$. So, its volume $(V) = \frac{1}{3}\pi r^2 h = \pi(4)^2(2) = \frac{32}{3}\pi$

C) $2\pi r^2 = 2\pi rh$ and $h = 2 \Rightarrow r = 2$. So, its volume $(V) = \pi r^2 h = \pi(2)^2(2) = 8\pi$

D) $\pi r^2 = 2\pi rh$ and $h = 2 \Rightarrow r = 4$. So, its volume $(V) = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(4)^2(2) = 16\pi$

11. How many **edges** does an oblique pentagonal pyramid have?

A) 6 B) 10 C) 5 D) 8

The reason for your answer above is

A) Since the base of the pyramid is a pentagon (i.e. five-sided polygon) and its faces are triangles (i.e. three-sided polygon), the number of edges of the pyramid becomes 8.

B) Since the base of the pyramid is a pentagon (i.e. five-sided polygon) and its faces are triangles that meet at one point, the number of edges of the pyramid becomes 6.

C) Since the base of the pyramid is a pentagon (i.e. five-sided polygon), the number of edges of the pyramid becomes 5.

D) Since the base of the pyramid is a pentagon (i.e. five-sided polygon) and its faces are triangles that have five common sides, the number of edges of the pyramid becomes 10.

12. A triangular pyramid and a triangular prism have the **same** base and height. How many times the volume of the *prism* is greater than the volume of the *pyramid*?

A) 2 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) 3

The reason for your answer above is

Let $\boldsymbol{V_1}$ be the volume of the prism and $\boldsymbol{V_2}$ be the volume of the pyramid. Then,

A) $V_1 =$ (Base area) (height) and $V_2 = \frac{1}{3}$(base area) (height) $\Rightarrow V_1 = 3V_2$

B) $V_1 =$ (Base area) (height) and $V_2 = \frac{1}{2}$(base area) (height) $\Rightarrow V_1 = 2V_2$

C) V_2 = (Base area) (height) and $V_1 = \frac{1}{3}$ (base area) (height) ⇒ $V_1 = \frac{1}{3}V_2$

D) V_2 = (Base area) (height) and $V_1 = \frac{1}{2}$ (base area) (height) ⇒ $V_1 = \frac{1}{2}V_2$

13. If each edge of a regular tetrahedron is 12 cm, then its *lateral surface area* is _______

A) $216\sqrt{3}\ cm^2$ B) $108\sqrt{3}cm^2$ C) $144\sqrt{3}cm^2$ D) $288\sqrt{3}cm^2$

The reason for your answer above is

All faces of a tetrahedron are equilateral triangles whose edge is *s* =12 cm long. So,

A) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{4} = \frac{(12\ cm)^2\sqrt{3}}{4} = \frac{144\sqrt{3}cm^2}{4} = 36\sqrt{3}cm^2$ ⇒its lateral surface area becomes $4(36\sqrt{3}cm^2) = 144\sqrt{3}cm^2$

B) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{2} = \frac{(12\ cm)^2\sqrt{3}}{2} = \frac{144\sqrt{3}cm^2}{2} = 72\sqrt{3}cm^2$ ⇒its lateral surface area becomes $3(72\sqrt{3}cm^2) = 216\sqrt{3}cm^2$

C) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{4} = \frac{(12\ cm)^2\sqrt{3}}{4} = \frac{144\sqrt{3}cm^2}{4} = 36\sqrt{3}cm^2$ ⇒its lateral surface area becomes $3(36\sqrt{3}cm^2) = 108\sqrt{3}cm^2$

D) The area (A) of one triangle is $\frac{s^2\sqrt{3}}{2} = \frac{(12\ cm)^2\sqrt{3}}{2} = \frac{144\sqrt{3}cm^2}{2} = 72\sqrt{3}cm^2$ ⇒its lateral surface area becomes $4(72\sqrt{3}cm^2) = 288\sqrt{3}cm^2$

14. The *diameter* of the Moon is approximately **one-fourth** of the *diameter* of the Earth. What fraction of the volume of the Earth is the volume of the Moon?

A) $\frac{1}{64}$ B) $\frac{1}{512}$ C) $\frac{1}{16}$ D) $\frac{1}{12}$

The reason for your answer above is

Let *r* = radius of Moon; *R* = radius of Earth; V_{Moon} is the volume of the Moon and V_{Earth} is the volume of the Earth. Then,

A) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{1}{4}R)^3 = \frac{1}{12}[\frac{4}{3}\pi R^3] = \frac{1}{12}V_{Earth}$

B) $r = \frac{1}{8}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(\frac{1}{8}R)^3 = \frac{1}{512}[\frac{4}{3}\pi R^3] = \frac{1}{512}V_{Earth}$

C) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{1}{4}R)^3 = \frac{1}{64}[\frac{4}{3}\pi R^3] = \frac{1}{64}V_{Earth}$

D) $r = \frac{1}{4}R \Rightarrow V_{Moon} = \frac{4}{3}\pi r^2 = \frac{4}{3}\pi(\frac{1}{4}R)^2 = \frac{1}{16}[\frac{4}{3}\pi R^2] = \frac{1}{16}V_{Earth}$

15. What is the volume of a right cone with a base diameter of 21 cm and height of 4 cm?

A) $441\pi cm^3$ B) $294\pi\ cm^3$ C) $220.5\pi\ cm^3$ D) $147\pi cm^3$

The reason for your answer above is

Given: diameter (***d***) = 21 m & height (***h***) = 4 m. Let ***V*** be the volume of the right cone. Then,

A) $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{21}{2}\ m\right)^2(4m) = \frac{1}{3}(21)^2\pi\ m^3 = \frac{1}{3}(441\pi\ m^3) = 147\pi\ m^3$

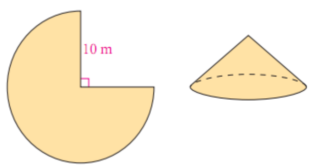
B) $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi\left(\frac{21}{2}\ m\right)^2(4m) = \frac{1}{2}(21)^2\pi\ m^3 = \frac{1}{2}(441\pi\ m^3) = 220.5\pi\ m^3$

C) $V = \frac{1}{4}\pi r^2 h = \frac{1}{4}\pi(21\ m)^2(4m) = (21)^2\pi\ m^3 = 147\pi\ m^3$

D) $V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi\left(\frac{21}{2}\ m\right)^2(4m) = 21^2\pi\ m^3 = 441\pi\ m^3$

16. A cone is formed from a sector of a disk that has a radius of 10 cm as you see in the side figure. What is the *lateral surface area* of the cone?

A) $\frac{25}{4}\pi\ cm^2$ B) $\frac{75}{4}\pi\ cm^2$ C) $75\pi\ cm^2$ D) None of these



The reason for your answer above is

The lateral face area (LA) of the cone is equal to the area of the sector with radius(***r***) and central angle (***θ***). Thus,

A) $LA = \frac{\pi r^2 \theta}{360^0} = \frac{\pi(5\ cm)^2 270^0}{360^0} = \frac{75}{4}\pi\ cm^2$ B) $LA = \frac{\pi r^2 \theta}{360^0} = \frac{\pi(10\ cm)^2 270^0}{360^0} = 75\pi\ cm^2$

C) $LA = \frac{\pi r^2 \theta}{360^0} = \frac{\pi(5\ cm)^2 90^0}{360^0} = \frac{25}{4}\pi\ cm^2$ D) None of these

17. What is the surface area of the sphere if its volume is $\frac{32\pi}{3} cm^3$?

A) $32\pi\ cm^2$ B) $72\pi\ cm^2$ C) $12\pi\ cm^2$ D) $16\pi cm^2$

The reason for your answer above is

Let ***r***, ***SA***, and ***V*** be the radius, surface area, and volume of the sphere respectively. Then,

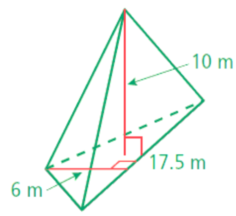
A)$V = \frac{4}{3}\pi r^3 = \frac{32\pi}{3} cm^3 \Rightarrow r^3 = 8\ cm^3 \Rightarrow r = 2\ cm.$ So, $SA = 4\pi r^2 = 4\pi(2\ cm)^2 = 16\pi\ cm^2$

B)$V = \frac{2}{3}\pi r^2 = \frac{32\pi}{3} cm^3 \Rightarrow r^2 = 12\ cm^2 \Rightarrow r = 4\ cm.$ So, $SA = 2\pi r^2 = 2\pi(4\ cm)^2 = 32\pi\ cm^2$

C)$V = \frac{1}{3}\pi r^3 = \frac{32\pi}{3} cm^3 \Rightarrow r^3 = 32\ cm^3 \Rightarrow r = 6\ cm.$ So, $SA = \frac{1}{3}\pi r^2 = \frac{1}{3}\pi(6\ cm)^2 = 12\pi\ cm^2$

D)$V = \frac{1}{3}\pi r^3 = \frac{32\pi}{3} cm^3 \Rightarrow r^3 = 32\ cm^3 \Rightarrow r = 6\ cm.$ So, $SA = 2\pi r^2 = 2\pi(6\ cm)^2 = 72\pi\ cm^2$

18. The volume of the right triangular pyramid shown to the side is ______



A) $175\ m^3$ B) $355\ m^3$ C) $725\ m^3$ D) None of these

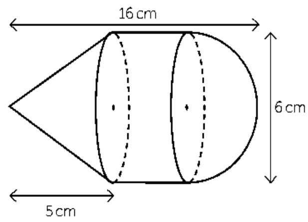
19. The *total surface area* of the rectangular prism with the dimensions 3cm, 4 cm, and 5 cm is ___?

A) 94 cm^2 B) 82 cm^2 C) 70 cm^2 D) 112 cm^2

20. The volumes of a cylinder and a sphere with equal radii ***r*** are **equal**. The **altitude** of the cylinder in terms of ***r*** is ______

A) $4r$ B) $2r$ C) $\frac{4r}{3}$ D) $\frac{r}{3}$

21. Find the volume of the following composed figure shown to the side.



A) $123\pi\ cm^3$ B) $153\pi\ cm^3$ C) $150\pi\ cm^3$ D) $114\pi\ cm^3$

22. If the total surface area of a regular square pyramid is 144 cm^2 and the length of one side of its base is 8 cm, then the volume of the pyramid is ______

A) 8 B) 64 C) 80 D) 106.6

23. A cone has a volume of $600\pi cm^3$ and a height of 50 cm. What is the radius of the cone?

A) 3.5 cm B) 6.0 cm C) 10.6 cm D) 36.0 cm

24. A conical tent is 10 m high and the radius of its base is 24 m. The tent slant height is ___

A) 26m B) 28m C) 25m D) 27m

25. The lateral surface area of a cone is 308 cm^2 and its slant height is 14 cm. The radius of its base is___

A) $\frac{14}{\pi}$ cm B) $\frac{20}{\pi}$ cm C) $\frac{21}{\pi}$ cm D) $\frac{22}{\pi}$ cm

26. The total surface area of a hemisphere of radius 10 cm is______ [use $\pi = 3.14$]

A) 842 cm^2 B) 940 cm^2 C) 942 cm^2 D) 840 cm^2

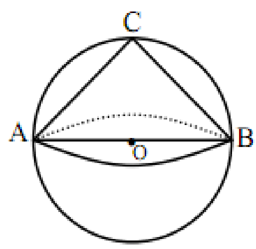
27. The surface area of a sphere with a diameter of 14 cm is____

A) 784π cm^2 B) $588\pi cm^2$ C) $392\pi cm^2$ D) $299\pi cm^2$

28. If the radius of a sphere is $\frac{2a}{3}$ cm, then its volume is _____

A) $\frac{32}{81}\pi a^3 cm^3$ B) $\frac{23}{4}\pi a^3 cm^3$ C) $\frac{32}{3}\pi a^3 cm^3$ D) $\frac{34}{3}\pi a^3 cm^3$

29. In the given figure, what is the **ratio** of the volume of the sphere to the volume of the cone is _______?



A) 2 B) $\frac{5}{2}$ C) $\frac{7}{2}$ D) 4

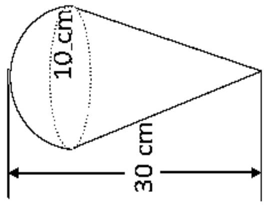
30. The base area of a regular square pyramid is 100 cm^2 and the sum of the area of its *lateral faces* is 260 cm^2. What is the *altitude* of the pyramid?

A) 8 cm B) 10 cm C) 12 cm D) 13 cm

31. What is the volume of a frustum of a cone with a height of 5 cm and the radii of its bases are 3 cm and 4 cm?

A) $\frac{295}{3}\pi\ cm^3$ B) $\frac{240}{3}\pi\ cm^3$ C) $\frac{105}{3}\pi\ cm^3$ D) $\frac{290}{3}\pi\ cm^3$

32. If ice cream consists of a hemisphere with a radius of 10 cm and a cone as shown in the side figure, then its volume is ________



A) $\frac{8000}{3}\pi\ cm^3$ B) $8000\pi\ cm^3$ C) $4000\pi\ cm^3$ D) $\frac{4000}{3}\pi\ cm^3$

33. A triangular prism has a height of 30cm. Its base is a right triangle with legs 10cm and 24cm. The volume of this prism is __________

A) $2000cm^3$ B) $3000cm^3$ C) $4000cm^3$ D) $6000cm^3$

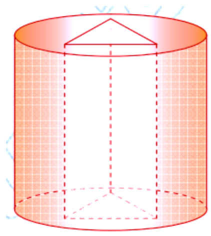
34. A frustum of a regular square pyramid has a height of 5 cm. The upper base is of side 2 cm and the lower base is of side 6 cm. *The lateral surface area* of the frustum is ______

A) $16\sqrt{29}\ cm^2$ B) $32\sqrt{21}\ cm^2$ C) $16\sqrt{27}\ cm^2$ D) $32\sqrt{27}\ cm^2$

35. The lower base of the frustum of a regular pyramid is a square 4 cm long; the upper base is 3 cm long. If the slant height is 6 cm, then its lateral surface area is _____

A) $24\ cm^2$ B) $12\ cm^2$ C) $21\ cm^2$ D) $18\ cm^2$

36. If a right circular cylinder whose base radius is 10 cm and whose height is 12 cm is drilled a triangular prism hole whose base has edges 3 cm, 4 cm, and 5 cm as shown below, then what is the total surface area of the remaining solid?



A) $(440\pi + 12)cm^2$ B) $(440\pi + 132)cm^2$

C) $(440\pi - 12)cm^2$ D) $(440\pi - 132)cm^2$

ANOVA Assumptions Test Results

The normality of MAPreTS was assessed using Shapiro-Wilk tests. For the EG1, the Shapiro-Wilk test was not significant, (W (31) = .964, p = .379). Similar results were found for EG2 (W (39) = .972, p = .427), and the CG (W (34) = .962, p = .280). These results suggest that the MAPreTS data were normally distributed for all study groups (see Table A1).

Table A1. Tests of Normality for MAPreTS

	Study Groups	**Shapiro-Wilk Statistic**	**df**	**Sig.**
MAPreTS	EG1	.964	31	.379
	EG2	.972	39	.427
	CG	.962	34	.280

Levene's Test for homogeneity of variance indicated no significant difference across the three study groups, (F (2,101) = .611, p =.545). This suggests that the assumption of homogeneity of variances was met for MAPreTS.

Table A2. ANOVA Result of MAPreTS

	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	251.393	2	125.697	12.816	.000
Within Groups	990.597	101	9.808		
Total	1241.990	103			

ANCOVA Assumptions Test Results

Normality tests for MAPostTS showed that the Shapiro-Wilk tests were not significant for EG1, (D(30) = .111, p = .200); EG2, (D (37) = .114, p = .200), and CG, (D (32) = .139, p = .118), confirming the normality of the data across the study groups (see Table A3).

Table A3. Tests of Normality for MAPostTS

	Study Groups	Shapiro-Wilk Statistic	df	Sig.
MAPostTS	EG1	.969	30	.507
	EG2	.957	37	.161
	CG	.947	32	.117

Levene's Test for Equality of Error Variances was not significant (F (2, 96) = 1.406, p = .250), indicating that the variances in MAPostTS were equal across the study groups (see Table A4).

Table A4. Levene's Test for Equality of Error Variances of MAPostTS

F	df1	df2	Sig.
1.406	2	96	.250
a. Design: Intercept + Groups + MAPreTS + Groups * MAPreTS			

As shown in Table A5 below, the interaction between groups and MAPreTS was significant, (F (2, 93) = 8.453, p = .000, η^2= .154), suggesting that the relationship between MAPreTS and MAPostTS differed across the groups.

Table A5. Tests of Between-Subjects Effects of MAPostTS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	661.657	5	132.331	29.858	.000	.616
Intercept	685.519	1	685.519	154.673	.000	.625
Groups	43.842	2	21.921	4.946	.009	.096
MAPreTS	161.617	1	161.617	36.465	.000	.282
Groups * MAPreTS	74.928	2	37.464	8.453	.000	.154
Error	412.181	93	4.432			
Total	11054.000	99				
Corrected Total	1073.838	98				

ANCOVA also assumes that relationships between the dependent variable and each covariate should be linear (Pallant, 2016). It was checked by drawing a scatter plot.

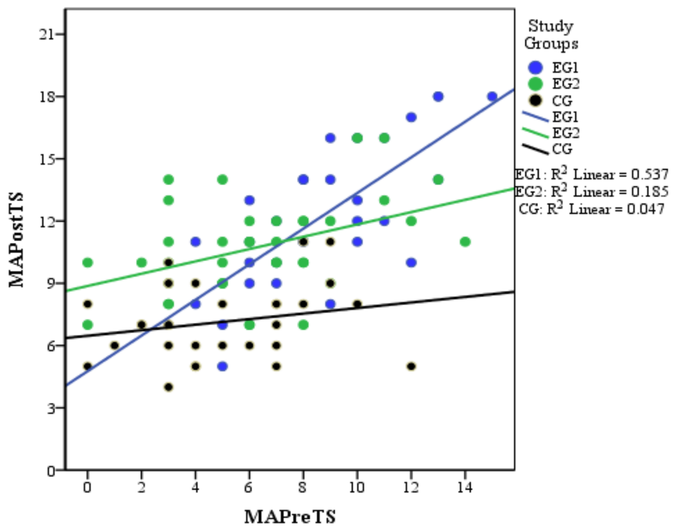


Figure A1. Linearity between MAPreTS and MAPostTS

The straight lines in Figure A1 indicate the linear relationship between the dependent variable (i.e., MAPostTS) and the covariate (i.e. MAPreTS) across the study groups.