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Flipped Classroom Model: Minimizing Gaps in Understanding Mathematical Concepts for Students with Different Academic Abilities

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Abstract: Each student has a different amount of time to fully understand information, students with high academic ability (UA) need less time than students with low academic ability (LA). Teachers should apply learning models that can facilitate their study time according to their individual needs. The aim of this research is to assess which learning model is most optimal in reducing the gap in understanding mathematical concepts between UA and LA students. Apart from that, this research also evaluates the effectiveness of implementing the flipped class (FC) model in increasing students' understanding of mathematical concepts, compared to the problem-based learning (PBL) model and conventional learning models. The research method used was the N-Gain Test and ANCOVA. The research results show that the FC model is the most optimal in reducing the gap in understanding mathematical concepts between LA and UA students. In addition, both FC and PBL models have proven effective in increasing students' understanding of mathematical concepts when compared to conventional models. Future research could consider combining the FC model with PBL or other learning models to see whether combining these models can improve students' understanding of mathematical concepts more significantly.

Keywords: *Academic abilities, flipped classroom, gaps in understanding concepts, problem-based learning.*

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Introduction

Understanding of students' mathematical concepts is important to note. It is difficult for students to understand mathematics without good conceptual knowledge, because the concepts in mathematics are interrelated. Understanding concepts is an important ability to make it easier for students to understand the next concept and is very important to help students solve more complex problems (National Council of Teachers of Mathematics [NCTM], 2000). Morgil and Yörük (2006) state that conceptual understanding is students' ability to detail what they have learned. Understanding concepts is one of the learning objectives that students must master (Ayas et al., 2010). Criteria for evaluating students' grasp of concepts may involve their capacity to articulate a concept in their own words, categorize objects based on specific properties related to the concept, express concepts through diverse mathematical representations, and apply concepts or problem-solving algorithms (Abraham et al., 1994). Students can solve more complex mathematical problems if they are able to understand the concept. This shows that understanding concepts in learning mathematics is a major milestone and helps students in solving the mathematical problems they face.

Every student possesses a unique pace in grasping the content presented by the teacher. Those with high academic abilities (UA) typically require a shorter duration to comprehend materials or concepts, in contrast to students with lower academic abilities (LA) (Prayitno & Suciati, 2017; Prayitno et al., 2022). Assigning identical study periods to all students without considering these ability differences may result in a disparity in understanding between LA and UA students. However, students with lower academic abilities can achieve mastery of the subject matter comparable to UA students if they receive study time tailored to their individual needs (Adeyemo & Babajide, 2014; Siddaiah-Subramanya et al., 2017). Consequently, it becomes crucial for educators to adopt learning models that facilitate optimal allocation of

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study time, ensuring each student can learn according to their unique levels of understanding. One effective learning strategy for achieving this objective is the flipped classroom (FC).

FC is a type of blended learning that changes the structure or traditional learning model (Bergmann & Sams, 2012; Flipped Learning Network, 2014; Staker & Horn, 2012). In this approach, learning materials are delivered outside of school through videos or other teaching materials, while material or concept deepening activities are carried out in class through discussion, problem solving, critical thinking, and project work with the support of feedback or instructor guidance. Subsequently, students are afforded the chance to enhance their understanding beyond the classroom setting through a sequence of assessments and evaluations (El Miedany, 2019; Francl, 2014; LaFee, 2013; Talbert, 2017). FC is grounded in the constructivism theory, providing students with the autonomy to independently cultivate their knowledge (Yen, 2020). As such, the FC approach can provide innovative ways to enhance student learning, leveraging technology and active participation to achieve deeper understanding.

The flipped classroom approach comprises three primary stages: pre-class, in-class, and post-class. In the pre-class stage, students are equipped with the necessary preparation to comprehend the material scheduled for discussion in the upcoming learning session. The main focus at this stage is so that students can remember and understand learning concepts. Furthermore, in class, students are given the opportunity to apply and analyze the material through various interactive activities. At this stage, the goal is to explore a deeper understanding. Finally, at the post-class stage, students are invited to evaluate and carry out project-based assignments as a follow-up after the learning session is completed (Ishartono, Nurcahyo, Waluyo, Prayitno, et al., 2022; Ishartono, Nurcahyo, Waluyo, Razak, et al., 2022; Roehling, 2018). Therefore, it can be concluded that the FC approach includes three distinct phases, which enable students to be actively involved in the learning process. Students use time outside of class to prepare and use time in class to apply the concepts they have acquired. This approach aims to increase student participation and strengthen their understanding of the learning material.

The flipped classroom (FC) approach gives students greater responsibility for their own learning process. Students become more proactive and have the flexibility to arrange study time and place according to their convenience. Apart from that, they can repeat the material if there are concepts they still don't understand (Angelone et al., 2020; Owston et al., 2013). The FC approach also encourages the development of more effective independent learning skills for students (Ishartono, Nurcahyo, Waluyo, Prayitno, et al., 2022; Suryawan et al., 2021). When facing difficulties, students are expected to find solutions and solve these problems independently. The autonomy cultivated through this learning process has the potential to enhance students' self-confidence in the classroom, thereby positively influencing their engagement in learning (Lin et al., 2019; Naibert et al., 2021; O'Flaherty & Phillips, 2015). Instructors can optimize class time by incorporating more engaging, interactive learning activities, or practical projects (Vaughan, 2014). The flipped classroom (FC) model affords educators increased opportunities to assess their students' grasp of the material and identify both strengths and weaknesses. In contrast to traditional classes, where instructors often focus on active and dominant students, FC allows for a more inclusive approach, ensuring that less active students also receive attention and recognition. With FC, educators can customize the content, processes, and products they want to produce according to the needs and characteristics of each student.

According to the literature spanning the last decade, multiple meta-analysis studies affirm the effectiveness of the problem-based learning (PBL) model in enhancing student learning achievements. PBL has been demonstrated to outperform conventional learning, as evidenced by studies conducted by Kong et al. (2014), Suparman et al. (2021), Susanti et al. (2020), Yew and Goh (2016), and Yunita et al. (2020). In the implementation of PBL, students are organized into groups for collaborative discussions, engaging in the exploration of non-routine questions through Student Worksheets (LKS). The teacher's role in PBL is that of a facilitator, offering support or scaffolding (Amalia et al., 2017; Michaelsen et al., 2014). PBL fosters the provision of scaffolding from both teachers and peers. Within the PBL framework, students with high academic abilities provide assistance to those with lower academic abilities, and simultaneously receive support from the teacher. The incorporation of scaffolding in PBL holds the potential to diminish the gap in understanding mathematical concepts between students with low and high academic abilities. However, to date, there has been no research that specifically investigates the effectiveness of PBL in overcoming gaps in conceptual understanding between students of various levels of academic ability. Therefore, this research aims to explore whether PBL can successfully overcome these problems.

In light of these objectives, the aim of this research is to assess which learning model is most optimal in reducing the gap in understanding mathematical concepts between students with upper academic ability (UA) and lower academic ability (LA). Apart from that, this research also evaluates the effectiveness of implementing the flipped classroom (FC) model in increasing students' understanding of mathematical concepts, compared to the problem-based learning (PBL) model and conventional learning models. This investigation is important to show that the proposed learning model is not only beneficial for students with high academic abilities but also benefits those with low academic abilities. Apart from that, this research can be a basis for educators and stakeholders in making the right decisions.

Methodology

Research Design

This research employs a 3 x 2 factorial design to assess the effectiveness of implementing the Flipped Classroom (FC), Problem-Based Learning (PBL), and conventional learning models in enhancing the comprehension of mathematical concepts. This design also aims to compare the effectiveness of the three models in minimizing the gap in understanding mathematical concepts between students with low (LA) and high (UA) academic abilities. This design schematic is outlined in Table 1.

Table 1: Research Design Scheme

Academic Ability	Learning Model		
	FC (A_1)	**PBL (A_2)**	**Conventional (A_3)**
Lower Academic (B1)	A_1B_1	A_2B_1	A_3B_1
Upper Academic (B2)	A_1B_2	A_2B_2	A_3B_2

Participant

This research focused on the entire population of seventh-grade students at SMPN 14 Ambon, Indonesia. Out of the available four classes, three were chosen randomly to be part of this study. The sample size comprised 150 students, evenly distributed across three groups. Each group included 25 students with low academic abilities (LA) and 25 students with high academic abilities (UA). Prior to the commencement of the research, students were briefed on the research objectives and voluntarily agreed to participate. The research was conducted between March and May 2023.

Instruments and Procedures

This research applies a post-test as a tool to measure students' conceptual understanding on the topic of algebraic operations. The pre-test and post-test each consist of six multiple choice questions. To avoid bias, two mathematics expert lecturers reviewed and validated the questions and answers. The reliability coefficient of multiple choice questions reached 0.78, indicating a high level of reliability. The experimental process consists of three main stages: pre-test, experimental procedure, and post-test. In the first step, a pre-test was used to ensure the equality of the groups. Then, in the second stage, each group took a course for six weeks, as illustrated in Figure 1. The research encompassed three distinct sample groups, each subjected to a different treatment: the Flipped Classroom (FC), Problem-Based Learning (PBL), and conventional learning. The syntax of the FC and PBL models employed in this study are elucidated in Figure 2.

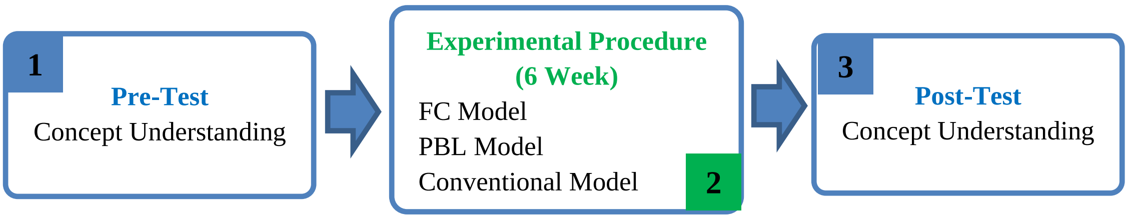

Figure 1. Experimental Procedure

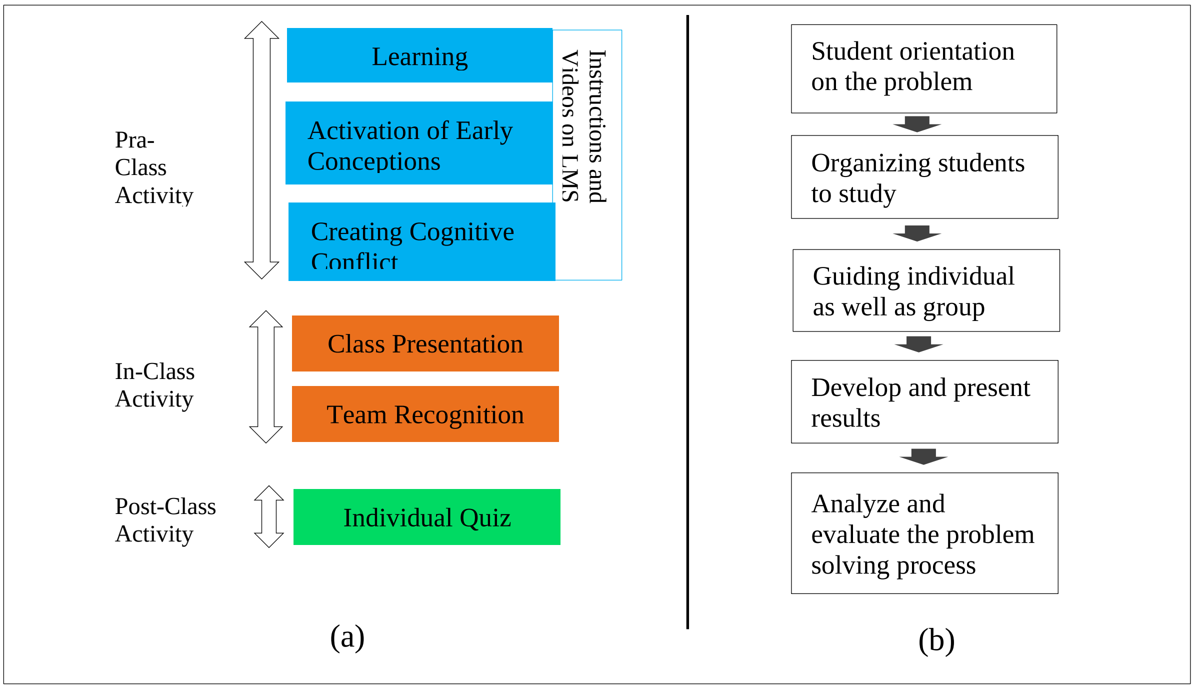

Figure 2. Stages of FC Model (a) and PBL Model (b)

Data Analysis

The data analysis in this study employed the N-Gain and ANCOVA tests, utilizing pretest scores as a covariate. The N-Gain test was employed to assess the increase in understanding of mathematical concepts among LA and UA students following the implementation of the FC, PBL, and conventional learning models. Effectiveness levels were categorized according to Hake's (2002) classification, where N-Gain values exceeding 76% were labeled as "Effective," values ranging from 56% to 76% as "Moderately Effective," values between 40% and 56% as "Less Effective," and values below 40% as "Ineffective."

ANCOVA was carried out to evaluate the relative effectiveness of the three models in reducing the gap in conceptual understanding between LA and UA students. Prior to the ANCOVA analysis, normality and data homogeneity tests were conducted. The Kolmogorov-Smirnov normality test yielded values of 0.186 for the pre-test and 0.179 for the post-test, both exceeding 0.05, indicating a normal distribution of data. The Levene's test for homogeneity produced a significance value of 0.069, surpassing 0.05, suggesting homogeneity of variance across data groups. Tukey's post-hoc test was employed to assess significant differences in mean values of variables. The statistical analysis was performed using IBM SPSS software at a confidence level of 0.05.

Results

Effectiveness of the FC Model on LA and UA Students' Understanding of Concepts

The analysis phase commences by categorizing data derived from the assessment of students' comprehension of mathematical concepts. Subsequently, the average pretest and posttest scores of the students were computed utilizing the Average N-Gain formula. The N-Gain test outcomes are presented in Table 2 below.

Table 2. Average Pre-Test, Post-Test, Standard and N-Gain Learning Achievement Results

Learning Model	**Academic Abilities**	**N**	**Pretest**		**Posttest**		**N-Gain**
			M	**SD**	**M**	**SD**	
Conventional	LA	25	29.81	8.65	50.95	8.20	30.12%
	UA	25	60.42	7.43	83.33	7.75	48.25%
	Total	50	44.50	17.50	66.49	18.27	36.32%
PBL	LA	25	26.28	6.45	61.54	8.36	50.93%
	UA	25	57.99	7.42	86.29	7.35	73.44%
	Total	50	41.50	17.53	73.42	14.80	58.57%
FC	LA	25	30.77	7.33	64.10	7.32	72.22%
	UA	25	61.46	7.56	92.01	5.75	63.96%
	Total	50	45.50	17.26	77.50	15.64	69.42%

Table 2 indicates that the overall N-Gain values for concept understanding, when applying the conventional model, are 36.32% (Not Effective), for the PBL model are 58.57% (Quite Effective), and for the FC model are 69.42% (Quite Effective). This suggests that the comprehension of mathematical concepts is notably effective when utilizing the FC and PBL models but is not effective when employing the conventional model. The analysis results also reveal that the comprehension of mathematical concepts is quite effective for both LA and UA students when applying the FC model. In contrast, in the PBL model, the understanding of mathematical concepts is quite effective for UA students but not effective for LA students. When implementing the conventional model, the analysis indicates that the understanding of mathematical concepts by both UA and LA students is not effective.

The subsequent step involves conducting the ANCOVA test to assess students' conceptual understanding based on three learning model treatments, academic abilities, and the interaction between the two. The outcomes of this test are presented in Table 3.

Table 3. Effect Of Learning Strategies, Academic Level, And Interactions On Conceptual Understanding

Source	Sum of Squares	Df	Mean Square	F	Sig.
Corrected value of Model	20337.084[a]	6	3389.514	662.343	0.000
Intercept	1779.365	1	1779.365	347.705	0.000
Average Score	3558.870	1	3558.870	695.437	0.000
Learning approaches	1591.730	2	795.865	155.520	0.000
Academic Abilities	5.852	1	5.852	1.144	0.289
Learning Model*Academic Abilities	235.177	2	117.589	22.978	0.000
Error	347.987	143	5.117		
Total	414583.436	150			
Total Average Score	20685.071	149			

After controlling for students' initial conceptual understanding, the two-way ANCOVA test results (refer to Table 3) demonstrate significant differences in students' conceptual understanding based on different learning models [$F(2,145) = 155.52$, $p < 0.05$]. Subsequently, a post-hoc test was conducted, and the results are presented in Table 4.

Table 4. Post-Hoc Test Results For Differences In Students' Conceptual Understanding

Learning Models	Pretest	Post-Test	SD	Difference	Corrected Mean	Notation
Conventional	44.50	66.49	16.55	20.16	65.92	A
PBL	41.50	73.42	14.80	31.92	75.54	B
FC	45.50	77.50	6.91	37.83	75.88	B

The results of the post-hoc test (refer to Table 4) reveal that there is no significant difference between the FC and PBL models (indicated by the notation "b"), while both the FC and PBL models significantly differ from the conventional model (indicated by the notation "a"). These findings suggest that the implementation of both the FC and PBL models is more effective when compared to conventional models.

Comparison of the Effectiveness of FC, PBL, and Conventional Models in Minimizing Conceptual Understanding Gaps between LA and UA Students

To further assess the effectiveness of the three models, a detailed analysis was conducted using Tukey's post hoc test. The outcomes of the preceding ANCOVA analysis (refer to Table 3) indicate a significant difference in the interaction between the learning model and students' academic abilities. In simpler terms, there exists a noteworthy interaction effect between the learning model and students' academic abilities concerning their understanding of mathematical concepts. A concise summary of the post-hoc test results is provided in Table 5 below.

Table 5. Variations In The Comprehension Of Concepts Among Students From LA And UA.

Learning Models	Academic Abilities	Pretest	Post-Test	Difference	Corrected Mean	Notation
Conventional	LA	29.81	50.95	21.14	64.37	a
Conventional	UA	60.42	83.33	22.91	67.46	ab
PBL	UA	57.99	86.29	28.30	72.74	bc
FC	UA	61.46	92.01	30.55	75.14	cd
FC	LA	30.77	64.10	33.33	76.61	d
PBL	LA	26.28	61.54	35.26	78.34	d

According to the results from Table 5, the post-hoc tests reveal the following: 1) Notable variations exist in the comprehension of mathematical concepts between students with low academic abilities (LA) and those with high academic abilities (UA) who underwent treatment with the conventional model (indicated by notations a and b); 2) A significant difference in the understanding of mathematical concepts is observed between UA students treated with the conventional model and those receiving Problem-Based Learning (PBL) treatment (indicated by notations b and c); 3) A significant difference is noted in the understanding of mathematical concepts between UA students undergoing PBL treatment and those undergoing Flipped Classroom (FC) treatment (indicated by notations c and d); and 4) No significant difference is found in the understanding of mathematical concepts between LA students undergoing FC treatment and UA students undergoing PBL treatment (indicated by notation d).

In summary, these findings lead to the conclusion that the Flipped Classroom (FC) model is more effective in bridging the gap in understanding mathematical concepts between students with low (LA) and high (UA) academic abilities compared to the Problem-Based Learning (PBL) model and conventional learning.

Discussion

Effectiveness of the FC Model on LA and UA Students' Understanding of Mathematical Concepts

In general (without considering academic ability), our findings indicate the effectiveness of the Flipped Classroom (FC) and Problem-Based Learning (PBL) models in enhancing students' understanding of mathematical concepts. Evaluation based on the N-Gain score classifies the FC and PBL groups as "Moderately Effective," while the conventional model group is deemed "Ineffective." These results align with prior studies conducted by Chen et al. (2018), Gillette et al. (2018), Låg and Sæle (2019), Lo and Hew (2019), Lozano-Lozano et al. (2020), Shi et al. (2020), and Purnomo et al. (2022), which assert that the FC model can enhance student learning outcomes compared to conventional methods. Additionally, our research supports the findings of previous studies by Kong et al. (2014), Suparman et al. (2021), Susanti et al. (2020), Yew and Goh (2016), and Yunita et al. (2020), demonstrating the effectiveness of PBL in improving learning achievements compared to conventional approaches.

If analyzed more deeply, research findings show that there is no significant difference in understanding mathematical concepts between students who follow the FC and PBL models. However, both showed significant differences when compared with the group that received conventional learning. These results indicate that FC and PBL are more effective in increasing understanding of mathematical concepts than conventional learning methods. However, when viewed from the perspective of academic ability, the application of the PBL model has proven to be quite effective in increasing conceptual understanding of high-performing students (UA), but less effective for low-performing students (LA). This is caused by limited student learning time in the PBL collaborative learning framework, which only provides guidance from UA students or educators to LA students during class time. These results are in line with research by Paristiowati et al. (2018), which states that collaborative learning can be more effective if given maximum time. In addition, PBL is unable to accommodate diverse learning needs, so LA students experience difficulty in adapting to their learning preferences, including learning style, time, and environment. In fact, individual needs to understand material or concepts as a whole can vary (Prayitno & Suciati, 2017; Prayitno et al., 2022). Low-performing students can achieve the same understanding of material as high-performing students if given study time that suits their needs (Siddaiah-Subramanya et al., 2017). Hence, our research concludes that the efficacy of the Problem-Based Learning (PBL) model is apparent primarily among high-performing students, while it proves to be less effective for those with lower academic performance.

In contrast to the PBL model, the results of our research show that the application of the FC model has proven to be quite effective in increasing LA and UA students' understanding of mathematical concepts. The FC model facilitates students' diverse learning needs, they can set the most comfortable time or place to study, and can also repeat it again if there is material they still don't understand. At FC, LA students have maximum study time to understand material or concepts outside of class. They also have plenty of time to plan problem-solving. Opportunity to ask questions if there is material that is not understood. In out-of-class activities (Pre-class activity), giving scaffolding by UA students to LA students can run optimally. In contrast to the PBL model which facilitates the provision of scaffolding in a shorter time. This is because collaborative learning is only done when learning in class.

In the context of this research, the FC model is proven to be more effective because it has characteristics that are able to accelerate the formation of the concept of balance in an individual's cognitive structure. If students already have a fairly complete general understanding of a previous concept, the assimilation and accommodation process can be accelerated. This means that there is an acceleration in the formation of conceptual balance in students' cognitive structures before classroom learning begins (Busyairi et al., 2022). In building understanding, students need to match the perceptions and objects studied with the initial schema in their cognitive structure. If perceptions and experiences are in line with the initial schema, the assimilation process tends to lead to the development of a more mature schema (Bächtold, 2013; DeRobertis, 2021). However, if the initial schema does not match the new experience, an imbalance may occur in the cognitive structure. Students must then make a decision whether the initial scheme needs to be replaced with a new one or retained. If the initial schema is replaced, students actively try to remember and strengthen the concepts they have to understand new concepts (Bormanaki & Khoshhal, 2017; Stoltz, 2018).

In the context of pre-class activities, students are equipped with preliminary knowledge or frameworks before participating in collaborative learning during class sessions. This implies that students possess an initial understanding of the upcoming material before attending the in-class learning session. This preparation fosters a heightened readiness among students to actively engage in discussions and collaborate within teams to address challenges, all of which occurs under the guidance of the teacher during class (Rau et al., 2017). A parallel discovery was made by Ishartono, Nurcahyo, Waluyo, Prayitno, et al. (2022), who found that assigning tasks and providing instructional materials to students before the class (pre-class) enhances students' prior knowledge, thereby better preparing them for active participation in classroom learning. Consequently, the effective understanding of mathematical concepts by both low academic ability (LA) and high academic ability (UA) students is achieved when the Flipped Classroom (FC) model is implemented.

Effectiveness of FC Strategy Implementation in Minimizing Concept Understanding Gaps Between LA and UA Students

Our research outcomes additionally indicate that there is no noteworthy distinction in the comprehension of mathematical concepts between students with low (LA) and high (UA) levels of understanding when implementing the Flipped Classroom (FC) model. In contrast, substantial differences are evident when employing the Problem-Based Learning (PBL) model and traditional teaching methods. This suggests that the FC model holds the potential to diminish the gap in understanding mathematical concepts between LA and UA students, while this does not happen in the PBL and conventional models. Although FC and PBL are both based on collaborative learning, The Flipped Classroom (FC) model has different characteristics that accommodate diverse learning needs, including variations in learning speed, learning style, and learning environment. The application of the FC model is seen as the right solution to overcome the gap in understanding mathematical concepts between students with lower academic ability (LA) and students with upper academic ability (UA). This suggests that a learning approach that is more focused on individual needs, as the FC model does, can have a positive impact in creating a learning environment that is inclusive and supportive of all students, regardless of their initial level of understanding. LA students can achieve an understanding that is on par with UA students if they are assisted to enter their potential development zone through scaffolding by teachers and peer tutorials (Prayitno et al., 2013). As previously explained, students with a low level of understanding (LA) have the potential to master the subject matter as well as students with a high level of understanding (UA) as long as they are given study time that suits their needs (Adeyemo & Babajide, 2014). Furthermore, by implementing the FC model, UA students have the opportunity to provide support (scaffolding) to LA students, helping them to move from the actual zone to the zone of proximal development. This approach is consistent with the views of (Haataja et al., 2019; Hendarwati et al., 2021) which states that support (scaffolding) provided appropriately can facilitate LA students in moving towards their zone of proximal development.

The implementation of the PBL model in learning time is considered not to optimally provide full support (scaffolding), thus creating a gap in understanding of mathematical concepts between students with higher academic abilities and students with lower academic abilities. Meanwhile, in the conventional model, the teacher explains the concept first, followed by collaborative student discussions about problems related to the topic. Despite collaborative learning, this strategy is still considered teacher-centered, an approach that several studies have found to be associated with lower student achievement (Ayaz & Sekerci, 2015; Gunduz & Hursen, 2015; Tamur & Juandi, 2020). In contrast, research shows that the collaborative nature of conventional groups fails to provide full support (scaffolding) exactly when it is needed most, thereby preventing students from reaching their zone of proximal development. Therefore, the research results show that the conventional model has proven to be ineffective in overcoming the gap in conceptual understanding between students with lower academic ability (LA) and students with upper academic ability (UA).

In order to achieve optimal conceptual understanding for students with high academic ability (LA) and low academic ability (UA), research suggests that teachers apply learning principles that are constructive, collaborative, and able to facilitate diverse learning needs. An alternative solution proposed is the adoption of the flipped classroom approach. This model serves as a viable option for educators aiming to enhance the comprehension of mathematical concepts and narrow the gap in understanding between students with high and low academic abilities.

Conclusion

Our research findings lead to the conclusion that there is a significant difference in students' understanding of mathematical concepts when the Flipped Classroom (FC) and Problem-Based Learning (PBL) models are compared to conventional learning models. The effectiveness of achieving an overall understanding of mathematical concepts is observed only when FC and PBL are implemented, not with conventional models. When considering academic abilities, the application of the FC model proves to be quite effective in enhancing the understanding of mathematical concepts for both low academic ability (LA) and high academic ability (UA) students. On the other hand, the application of the PBL model is notably effective in enhancing the understanding of concepts for UA students but not as effective for LA students. Meanwhile, the conventional model is deemed ineffective in fostering understanding for both LA and UA students. LA and UA students' conceptual understanding did not differ significantly when the FC model was applied, while conceptual understanding between LA and UA students when the PBL and conventional models were applied was significantly different. These results conclude that the FC model can minimize conceptual understanding between LA and UA students, but not the PBL and conventional models.

Recommendations

The following are several recommendations for further research on the effectiveness of the flipped classroom model in minimizing gaps in students' understanding of mathematics concepts. This research initially involved 150 students who came from the same location. Future research can broaden the sample by including more students from various regions in Indonesia, with the aim of ensuring more valid results. Later studies may include students of various levels of education, from primary to secondary education, and even in university settings. This will enable more comprehensive experimental research to evaluate the effectiveness of the flipped classroom model in teaching contexts, including the relationship between the mathematics curriculum and various other subjects. In our research, the effectiveness of the flipped classroom model was assessed in relation to the understanding of mathematical concepts of upper and lower ability students. Future research may consider combining different learning models. For example, combining the FC model with PBL or with other learning models to see if combining these models can improve students' understanding of mathematical concepts more significantly. In addition to the learning model, further research can consider other factors that influence students' understanding of mathematical concepts. This can include factors such as parental support, student motivation, or the evaluation methods used in the class. In addition to the academic impact, further research can also evaluate the social and psychological impact of implementing certain learning models. This can include student motivation, increased self-confidence, and students' feelings towards learning mathematics. Future research may also consider investigating contextual factors that might influence the effectiveness of the FC and PBL models, such as class size, school environment, or the level of support provided by teachers in implementing these models.

Limitations

This research has several noteworthy limitations that should be acknowledged. Firstly, the limitations pertain to the sample size, it is crucial for future research to incorporate a larger sample to enhance the reliability of the results, particularly in the context of teaching and learning within a specific discipline. Secondly, another limitation concerns the duration of the experimental intervention. Future studies could address this limitation by considering a more extended intervention period, allowing for a more comprehensive evaluation of the effects over time. Using a six-week period might be considered a relatively short period for investigating the effectiveness of an instructional approach. Therefore, in future studies, it is advisable to involve longer-running pedagogical experiments to enable a more comprehensive evaluation of the effectiveness of the learning approaches tested. Third, limitations related to external validity. This research was only conducted in one location. Further research may consider involving a wider variety of locations or contexts to test whether the results hold up more generally. Fourth, limitations related to other variables that might influence the results. In pedagogical experiments, there are many factors that can influence the results other than the learning approach itself. For example, the quality of teacher teaching, student motivation, and the learning environment can play a role in the results obtained. Therefore, in future research, it is important to control for or consider these factors so that the results obtained can be more precisely caused by the learning approach tested.

Authorship Contribution Statement

Sulistyowati: Concept, research design, resource management, methodology implementation, investigation, data analysis, and manuscript preparation. Rohman: resource management, data analysis, and manuscript preparation. Hukom: research conceptualization, developing methodology, and collecting data. All authors have read and given approval to the final manuscript.

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