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# Using Interactive Presentations to Promote Mathematical Discourse

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**Abstract:** The current study investigated whether: (1) using an interactive presentation (IP) platform could affect the amount of usage of the practices of making orchestrating mathematical discourse- sequencing and connecting students' responses. (2) using an interactive presentation (IP) platform could affect the amount of narratives constructed by students. Fifty seventh-grade students participated in the study; those students were divided into control and experimental groups. Qualitative and quantitative analyses were performed based on voice recordings and field notes. The results revealed that the teacher using (IP) asked nearly three times more questions that connected students' responses (i.e., questions that involved valuing students' ideas, exploring students' answers, incorporating students' background knowledge, and encouraging student-to-student communication). We also saw that the students participated in the learning processes. The students in the experimental group presented three times as many narratives as those in the control group. We present several excerpts from the transcripts of the classroom discussions to illustrate our findings. Discussion of the implications and limitations of these results and make recommendations based on those results.

**Keywords:** *Formative assessment, Interactive presentation, Mathematical discourse, Technology and teaching.*

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## Introduction

Vector numbers are an important aspect of mathematical knowledge and are crucial for understanding a wide range of mathematical concepts, such as algebra and calculus. However, from our experience students often have difficulty understanding vector numbers and this can have a negative impact on their learning and future success in mathematics in the future. One of the most important things for a teacher is to understand the difficulties that their students face and to be able to assess their abilities in order to provide appropriate support and guidance. The use of digital tools in the teaching process can fulfill this need by providing teachers with immediate feedback and helping them to identify deal and correct errors in the understanding of complex concepts such as vector numbers. The use of these tools can help to promote a more interactive and engaging learning environment, which can increase student motivation and engagement.

Students often have trouble with arithmetic operations involving vector numbers. Students have been found to have difficulty representing negative numbers. These difficulties lie in the numerical system, its direction, and the meaning of arithmetic operations (Altiparmak & Özdoğan, 2010; Ball, 1993; Booth & Siegler, 2006). The discourse method is considered an important teaching strategy and, today, researchers consider it to be a very important tool for formative assessment and overcoming difficulties, errors and misconceptions (Davis, 1997; Hansen et al., 2020; Sfard, 2007; Upton & Cohen, 2009; Wagganer, 2015). It should be noted that technological tools for the teaching process have been developed, including DGEs (Dynamic Geometry Environment) and assessment tools (Baya'a et al., 2017; Drijvers, 2013; Yerushalmy & Olsher, 2020). One of these tools is the Nearpod interactive presentation (IP), by which the researchers mean an educational tool that allows for effective collaborative learning between teachers and students. The teacher can show slides to the students during simultaneous instruction or make the slides available to students to study on their own after the lesson. Students can follow along with the presentation on their individual computers, add their own notes and respond to some assignments (Jelemenská et al., 2011). In the research literature, the researchers have not found

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any direct reference to the effects of the use of interactive presentations on classroom mathematical discourse. The current study is an attempt to fill this gap.

## Literature Review

*Mathematical discourse*

Upton and Cohen (2009) defined discussion as a process of communication between people. Social learning is built with the help of human contact and discussion contributes to the crystallization and enrichment of social learning. Communication is a routine social activity that includes a set of logical processes at the individual level and mathematical discourse refers to the totality of communication activities in the social environment.

One component of the term commognition that Sfard (2008) coined is narratives. She defined narratives as sentences describing objects or connections between objects or between processes that one can accept as correct or reject. The manner in which a statement is obtained is subject to the rules of mathematical discourse. For example, all mathematical theorems, definitions, and proofs are examples of accepted statements.

Through mathematical discourse, students are encouraged to share their understanding of mathematical concepts and participate fully in questioning, guessing, identifying, and explaining, which expands and enriches their circle of ideas on the subject of mathematics and makes it easier for them to progress in their learning (Brown, 2010; Chapin et al., 2003; Kersaint, 2015). Classroom discourse provides opportunities for students to use new mathematical vocabulary to understand their thinking and to train themselves. One practice of the teacher mentioned by O'Connor and Michaels (2019) is revoicing which mean representing student answer as it without evaluating them, they reported the impact of this practice on acquiring more accurate mathematical language.

Wagganer (2015) wrote about four useful strategies that teachers can use to support and develop mathematical discourse: (a) speaking with students about the importance of mathematical discussion and the importance of listening to others and how to respond, (b) presenting the main sentences that students mention regarding the topic at hand, (c) focusing on the comparison of the explanation and the justification process, and (d) giving examples of positions.

Stein et al. (2008) proposed a pedagogical model which includes five practices in order to orchestrate mathematical discourse: anticipating students' responses when constructing mathematical tasks teachers expect the learners to interpret the problem mathematically. Monitoring students' responses is done by paying attention to students thinking when circulating among the learners. Selecting students' responses for public display after monitoring the available students' responses. Sequencing selected students' responses when organizing the student's presentation sequence and connecting students' responses when helping students to make judgments and draw connections between various ideas and approaches. After monitoring, selecting, and sequencing the teacher needs to make the connection.

Jacobs et al. (2010) mentioned three components of noticing mathematical thinking: attending to mathematical thinking, interpreting these aspects of thinking, and responding to them. When we look at the five practices of Stein et al. (2008), we can see that monitoring students' responses are correlated with the attending skill of noticing ability; whereas selecting, sequencing, and connecting students' responses are correlated with the responding aspect of noticing ability. These noticing skills are essential for formative assessment, which is defined as processes of gathering information about students' knowledge, interpreting that information, and using it to modify teaching and learning activities (Black & Wiliam, 1998; Sadler, 1998).

White (2003) mentioned four themes that might help teachers make sequencing and connection to promote students' mathematical learning: (a) valuing students' ideas, (b) exploring students' answers, (c) incorporating students' background knowledge, and (d) encouraging student-to-student communication.

*Difficulties in understanding vector numbers*

In primary school, students are exposed to addition, subtraction, multiplication, and division. As pupils progress through elementary school, they explore other uses of less prominent representations (Harries & Tennant, 2012). Previous researchers have noted that students have difficulties dealing with vector numbers, especially estimating the value of vector numbers in real-life situations (Prather & Alibali, 2011). Altiparmak and Özdoğan (2010) have mentioned three categories of difficulties related to understanding vector numbers: (a) difficulties understanding the meaning of the numerical system and the direction of the number on the number axis, (b) the difficulties that students face with regard to arithmetic operations, and (c) difficulties related to subtraction operations. According to Hativa and Cohen (1995), the main difficulties students face can be divided into three types. First, there is a struggle between the actual meaning of the amount or quantity associated with the number in connection with the teaching of arithmetic in the early years of one's education and the concept of negative numbers. Second, there is a conflict between the two meanings of a minus symbol. Does it indicate a process or a sign? For example, in the statement: (-1) - (-2). Third, there is no good, intuitive, and appropriate model that fulfills all of the algebraic properties of vector numbers. Some pupils have a basic misunderstanding of the operations involved in vector numbers, especially with regard to negative numbers, since they sometimes ignore the sign indicating a negative number (Hayes & Stacey, 1990).

*Technology and teaching*

Even at early ages, students are more motived when they use educational platforms, which include screens, such as computers (Papadakis et al., 2018). To enhance students' learning, teachers should use technology as an aid, as opposed to an alternative solution (Drijvers, 2013; Martin, 2008). Aldon et al. (2017) mentioned a three-dimensional model that represents the different ways that students and teachers can use technology to enhance learning processes (Figure 1). The first dimension includes six strategies for analysis (Black & Wiliam, 2009): (a) clarifying and sharing learning intentions and success criteria, (b) the architecture of discussions, (c) effective classroom and other learning tasks that elicit evidence of student understanding, (d) providing feedback that helps students to progress in their learning, (e) activating students as learning resources for each other, and (f) activating students as the owners of their own learning. The second dimension includes the teacher, students, and peers. The third dimension includes ways in which technology can aid formative evaluation: (a) through data transmission and presentation, (b) through the processing and analysis of collected data, and (c) by providing an interactive environment in which students can work individually or collaboratively.

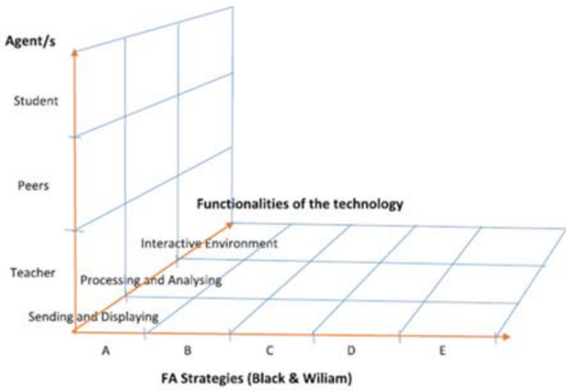


*Figure 1. Three-Dimensional Model Usage of Technology*

From Figure 1, we can clearly see that one function of technology is the interactive environment, defined as interactive learning based on an interactive process between the learner and the teacher that involves the use of modern and sophisticated techniques and tools (Beauchamp & Kennewell, 2010). Through interactive learning, learners have the opportunity to participate in activities that encourage them to think, comment on the material presented, and develop the skills of dealing with different concepts by analyzing information through discussion with others, asking questions, and expressing various opinions all of these skills lead the students to actively participate in the activities that take place within math class (Wegerif, 2007).

Empirical studies have shown that a technological online platform designed to help teachers assess students' knowledge through the use of example-eliciting tasks or inquiry-based learning affects teaching processes and enhances instruction. This can be attributed to the students' interactions with the given tasks and the feedback provided by the platform, which provides an opportunity to establish formative-assessment routines (Olsher et al., 2016; Popper & Yerushalmy, 2021). Haj-Yahya and Olsher (2022) revealed that the use of such platforms provides opportunities for pre-service teachers to develop their noticing skills. These tools are developed tools, which allow teachers to filter the students' responses not only by correctness but also by other pedagogical aspects such as the variance of examples, and it makes it easy to attend to and monitor students' mathematical thinking. The question arises whether less developed tools could make an impact also and how they could make the impact. McKay and Ravenna (2016) researched the tool of interactive presentation (IP) platform, which can be used to filter students' responses only according to whether or not they are correct. They found that using IP improved student participation and the evaluation of student performance, and also motivated students to learn during lessons.

A review of the literature shows very little research, which investigates in depth and in more qualitative methods, the impact of using interactive presentation (IP) on promoting mathematical discourse in the classroom. Mathematical discourse includes two connected components (among others): questions by teachers, which might sequence and connect students' responses, and narratives by students, which can be accepted or rejected by the teacher or by other students.

In the current study, we focused on these two components and investigated whether:

1. Using an interactive presentation (IP) platform could affect the amount of usage of the questions related to the fourth and fifth practices of sequencing and connecting students' responses.

2. Using the platform of interactive presentation (IP) affects the amount of students' participation with narratives during the mathematical discourse.

## Methodology

In the current study, the researchers used mixed methods. They recorded all of the lessons in a control group and an experimental group, to examine how the interactive environment affects the management of classroom discourse and the questions asked by the teacher and by students during the implementation of the intervention unit. These recordings also allowed the researchers to monitor the development of students' narratives in the two groups.

*Participants*

Fifty students from two seventh-grade classes participated in this study. Two classes were selected out of nine homogeneous classes taught at the same educational level; one served as the control group and the other was the experimental group. The researchers selected the two classes whose composition of students is the most heterogeneous in their math achievement in each 7th grade. This heterogeneity was confirmed by the fact that the students' grades on the vast majority of math tests were normally distributed (i.e., fit a bell curve) (Sartori, 2006). The teachers of the two classes each held a Bachelor's degree in Education and had 11–14 years of experience. The two teachers have a joint work plan and meet weekly to ensure that they present the same content. They also conduct joint tests for the two classes. The scores of the control and experimental group on three exams and the mathematical contexts examined by those exams are presented in Table 1. a Before conducting the study, observations were made on two lessons taught by the two teachers in the experimental and the control groups. It was found that there was no significant difference concerning the number of questions from each category coined by White (2003); less than three questions for all categories, once to the credit of the teacher of the experimental group and once to the credit of the teacher of the control group (see Table 1. b).

*Table 1.a. Mathematical Content of Three Consecutive Exams And Scores in The Control and Experimental Groups*

| **Test content** | **Average score, the control group** | **Average score, the experimental group** |
|---|---|---|
| Order of operations and the laws of arithmetic | 65% | 63% |
| Questions in the algebraic domain, writing an algebraic expression according to rules, writing an algebraic expression that describes, substitution in the algebraic expression | 58% | 55% |
| Writing an algebraic expression for a verbal problem, compiling similar terms for equivalent algebraic expressions, solving exercises involving the order of operations | 58% | 58% |

*Table 1.b. Question categories before IP intervention*

| **Types of questions asked** | **Control group** | **Experimental group** |
|---|---|---|
| Valuing students' ideas | 0 | 1 |
| Exploring students' answers | 3 | 5 |
| Incorporating students' background knowledge | 7 | 5 |
| Encouraging student-to-student communication | 5 | 4 |

The students in the experimental group participated in three lessons that involved the use of the platform of interactive presentation (IP) (Appendix 1). The control group participated in lessons that were the same, except that a whiteboard and marker were used instead of the platform of interactive presentation (IP). The two teachers used the same lesson plans, co-designed the lessons, wrote the same operative goals (i.e., the skills or knowledge the students would be expected to acquire), and wrote the expected difficulties that students might encounter in understanding the lesson and how those difficulties could be overcome. They used the same activities in the IP and whiteboard lessons agreed upon how they would respond to the expected wrong answers and coordinated all of their in-class moves. In the finding section, two out of those three lessons were analyzed. In both of the analyzed lessons, the students learned new things. These lessons investigate different situations of adding two numbers and the third lesson, which we will not report its findings is an extension of these lessons.

*Research tools*

The research tools used in the study included: (a) lessons involving interactive presentation (IP) for the experimental group, and (b) recorded lessons from the control and experimental groups.

In the IP platform, the teacher can include content slides or activity slides in the presentation, as shown in Figure 2. The content slides might be regular informative slides or show videos, web content, PDF files, etc. The activity slides might present open-ended questions, quizzes made up of multiple-choice questions, examples of drawings or segments, memory tests, matching pairs, etc. (see Figure 3).

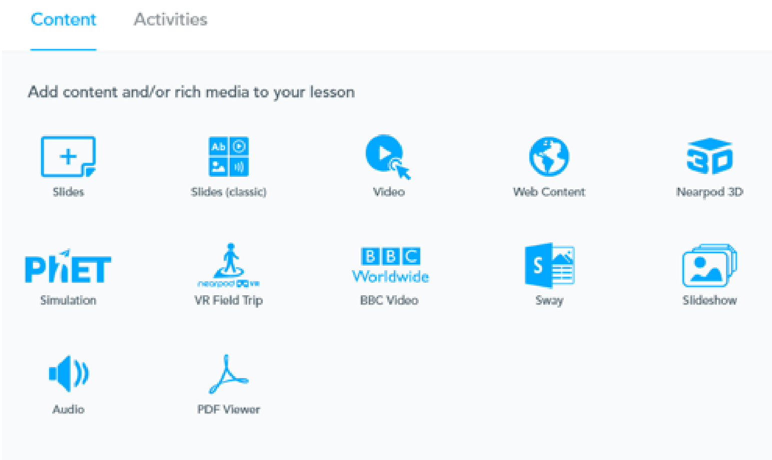


*Figure 2. Content Slides Options*

First, the teacher creates the presentation using a variety of different types of content, including slide shows, texts, quizzes, opn-ended questions, matching pairs, videos, and other activities. Second, the teacher shares the presentation with the students using a specific code that is automatically generated. Third, students log in and download the presentation. At this stage, depending on the teacher's decision, students will either experience a live session, in which they all progress through a slideshow while the teacher changes the slides, or interact with the content at their own pace.

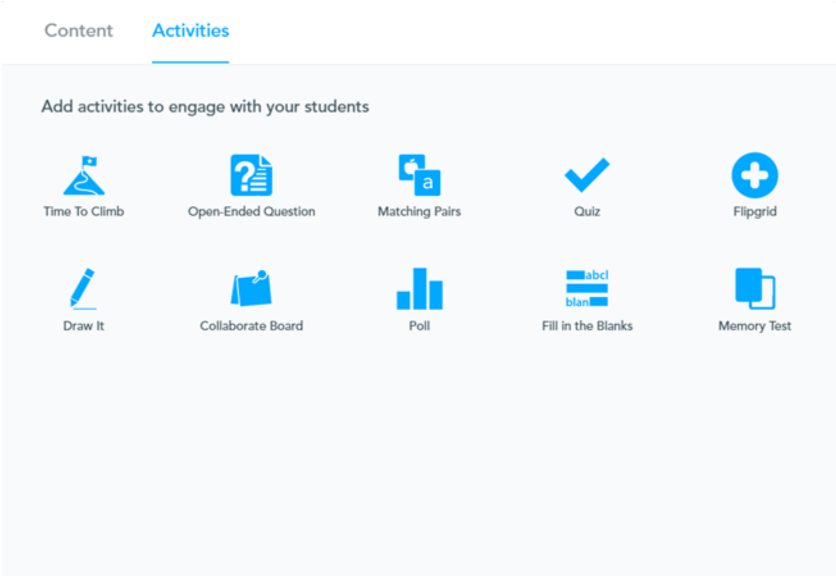


*Figure 3. Activities Slides Options*

By using activities slides students can actively participate during the lessons; this gives every student a chance to show what they know and allow the teacher to know where every student is in their learning. Automatic feedback involves answers constructed by the platform. (An example of such feedback for content slides, including quiz slides, is presented in Figure 4.) The student responses are arranged in rows, with wrong answers shown in red and correct answers shown in green. (The teacher defined the correct answer when he made the quiz slide.) When the slide contains an open-ended question, the students' answers are also arranged in rows, so the teacher can see the verbal responses. The teacher can see whether or not students have responded to an activity slide.

The activity slides that we used in our study were all quiz or drawing slides. It is important to note that quiz slides can be filtered according to correctness; whereas drawing slides cannot be filtered according to pedagogical factors, like the variability of examples (Haj-Yahya & Olsher, 2022; Mason, 2001; Olsher et al., 2016; Popper & Yerushalmy, 2021).

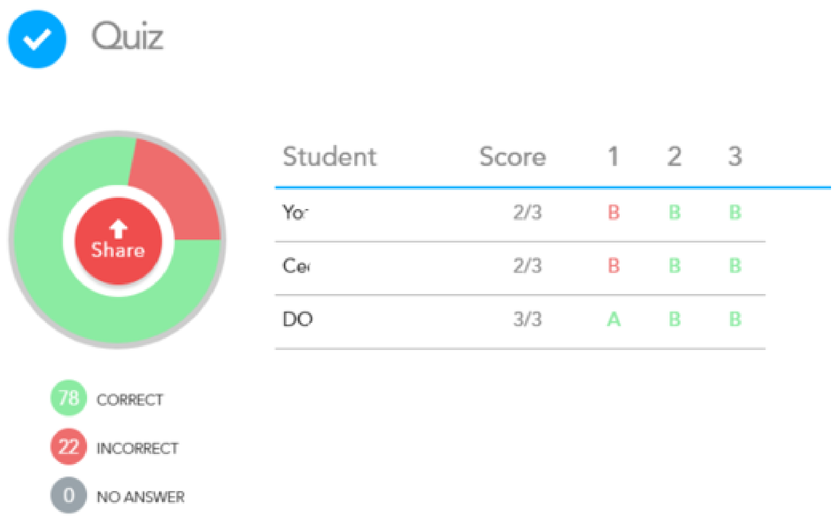


*Figure 4. Example of Constructed Feedback*

The five practices of Stein et al. (2008) are supported by the use of IP; when the teacher constructs the content or activity slides, the teacher activates the *anticipating* practice. Using automatic feedback reports (answers carpet) constructed by IP platform make it easy for the teachers to apply the rest four practices of Stein et al. (2008): *monitor* students' contribution, *select* students' response to display which is allowed by the share option (see Figure 4), organize the *sequence* of selected students' responses and after where to make the *connection* of the students' responses, the teacher is able to analyze the students' responses and discuss those responses with them (Hirtz, 2018).

*Data collection and analysis*

The researchers used mixed methods, including both qualitative and quantitative methods. The data-collection tool included audio recordings of three lessons in the control and experimental groups. We used directed content analysis by using categories used in existing prior research. We begin by identifying the categories derived from a theoretical perspective (Charmaz & Belgrave, 2007; Hsieh & Shannon, 2005). First, questions that were asked by the teachers were categorized and coded using deductive codes that we derived from White (2003). To ensure code reliability, code reviews were used, and two independent mathematics education researchers provide feedback on the code, after complete agreement on the code it implemented. In the quantitative analysis, all of the codes from the transcript were entered into an SPSS program, and frequencies were calculated. Second, the data from the two groups were analyzed according to narrative analysis (Bamberg, 2020; Sfard, 2008). The narratives developed by the teachers and the students in the two groups were counted and the frequencies of narratives constructed by students and those constructed by teachers were calculated.

## Findings

We analyzed the results according to the type of questions later we analyzed the results based on narratives.

*Analysis based on question categories*

*First lesson: Adding two numbers with the same sign*

*Table 2. Question Categories for the First Lesson: Adding Two Numbers With the Same Sign*

| Types of questions asked | Control group | Experimental group |
|---|---|---|
| Valuing students' ideas | 0 | 12 |
| Exploring students' answers | 4 | 16 |
| Incorporating students' background knowledge | 7 | 13 |
| Encouraging student-to-student communication | 6 | 15 |

During the first lesson, more questions that promote discourse were asked in the experimental group than in the control group (Table 2). The number of questions asked in the experimental group that related to sequencing and connecting practices of students' responses was more than three times the number of questions asked in the control group (56 questions in the experimental group and 17 in the control group). Sixteen questions that explored students' answers were asked in the experimental group, the largest number of questions that were asked, as compared to only four such questions in the control group. Fifteen questions that encourage communication between students were asked in the experimental group, as compared to six such questions in the control group. Similarly, 13 questions that integrated students' background knowledge were asked in the experimental group, as compared to seven such questions in the control group. Twelve questions that involved the assessment of students' ideas were asked in the experimental group; whereas no such questions were asked in the control group. The platform of interactive presentation gave the teacher more opportunities to select and to make sequences based on the student's responses which he received from the automatic feedback.

We will present several excerpts from the transcripts of the classroom discussions to illustrate our findings.

**Excerpt 1.** In the experimental group, the teacher asked: "What number must we add to the number (-2), in order to reach the point indicated in red? Write the addition exercise and explain your answer." The resulting discussion is reproduced below.

1. *Teacher: I want to choose one of the students' answers and show it to you (via the platform of interactive-presentation) Is the answer correct?*

*2. Students: Yes, it's right.*

*3. Teacher: You did well, and now we will see the student's explanation, "because if you subtract (-2) from (-10) we get (-8)." What do we mean?*

*4. Layan: Subtract two negative numbers.*

*5. Teacher: Your answer is wonderful. Can you explain?*

*6. Layan: I subtracted the larger number from the smaller number: meaning (-10) – (-20) = (-8).*

*7. Teacher: You took the large number minus the smaller number; what do you think?*

*8. Muhammad: The number negative 10 is smaller than negative 2.*

*9. Layan: Right, but I took them positive.*

*10. Teacher: How do we take the negative numbers of a positive? What is the process?*

*11. Students: Aaah, the absolute value.*

*12. Teacher: What do we mean by absolute value?*

*13. Muhammad: After the number, the steps from zero.*

*14. Teacher: Well done, the distance of the number from zero, which is always positive or zero.*

*15. Layan: Uh, I thought, in order to get from 2 to 10, we need to add the number 8 to the number 2, so that we see 8 + 2 = 10; 10-2 =8 … and then I turned them into negative numbers.*

*16. Teacher: Excellent. I want to explain so that we can find the number of steps, I took the difference between negative 10 and negative 2, or we talk about how far away 10 is from 2, you did the subtraction of vector numbers that are forward-looking, so we will learn it.*

*17. Samia: I did this: I asked how many steps away the number negative 2 is from the number negative 10, so I wrote 8 steps to the left, so the sum of negative 2 and negative 8 is negative 10.*

*18. Teacher: Well done, wonderful.*

*19. Ayman: Teacher, I made a mistake with my work when I made the end point, which is the number of steps ... so that it came out wrong.*

*20. Teacher: Well done, Ayman.*

In Excerpt 1, we can see that the automatic feedback constructed by the platform allowed the teacher to monitor and easily attend to the students' responses. She decided to select the right answer to share with the other student, in order to determine whether the students had based their answers on the correct reasoning. These steps allow teachers to use the fourth and fifth practices of sequencing and connecting students' responses. Four students participated in the discourse. The teacher used all four types of questions that might promote discourse in the classroom. In Line 1, she revoiced one student's answer and showed that she valued their ideas by asking them to notice the correctness of a response about a particular problem. In Line 5, we see an example of the teacher exploring the resources of students' answers. Specifically, the teacher wanted to know how the students arrived at their answers, even though those answers were correct. Layan explained how she solved the problem. In Line 7, we see the teacher encouraging interaction between students by asking the other students to judge Layan's response. After that question was asked, Muhammad joined in the discourse. In Line 12, the teacher asked a question that incorporated the students' background knowledge when she asked about the meaning of the absolute value. Muhammad was encouraged to answer. After the four types of questions were used, Samia also confirmed what had been said and Ayman found the error he had made in that same exercise. We can see obviously that after selecting student responses to display by the teacher, the students were encouraged to make the connections.

*Second Lesson: Adding Together Numbers That Have Different Signs*

*Table 3. Question Categories for The Second Lesson – Adding Together Numbers That Have Different Signs*

| Types of questions asked | Control group | Experimental group |
|---|---|---|
| Valuing students' ideas | 7 | 9 |
| Exploring students' answers | 0 | 17 |
| Incorporating students' background knowledge | 7 | 12 |
| Encouraging student-to-student communication | 3 | 12 |

Again, during the second lesson, the number of questions asked in the experimental group that promoted discourse related to sequencing and connecting practices of students' responses were about three times the number of such questions asked in the control group (Table 2). No questions exploring students' answers were asked in the control group. In contrast, 12 such questions were asked in the experimental group, accounting for the largest number of questions asked in that group. Nine questions evaluating students' ideas were asked in the experimental group, as compared to seven such questions in the control group. In the experimental group, 12 questions that integrated students' background knowledge were asked, as compared to only seven such questions in the control group. Similarly, 12 questions that encouraged student-to-student communication were asked in the experimental group, as compared to only three such questions in the control group.

Here are some excerpts from the experimental-group lesson.

**Excerpt 2.** The following question was presented in the platform of interactive presentation: "Solve $(+34)+(-48)$ without calculating the result of the exercise: Is it located to the right of zero or to the left of zero? Explain your answer!" The teacher presented the question and then presented the students' responses.

*1. Teacher: I want to write each answer on the board and discuss it. Who else wants to give me an answer?*

*2. Layan: Regarding the question, the first additive is 34 to the right of the zero on the number line, and (-48) is to the left of the zero. We go more steps to the left, meaning the answer seems to be less than zero.*

*3. Muhammad: I solved it in a different way, using the opposite number.*

*4. Teacher: Nice, but can you tell us how?*

*5. Muhammad: I wrote the opposite of number 34, which is -34, and put a dot roughly on it with the help of the number line. In order to reach -48 I have to go more steps in left side. Means the result less than zero.*

*6. Teacher: Muhammed is a logical thinker, so we will discuss this.*

*7. Jannah: It's easy if you think about it in everyday life. If I have 34 and I spent 48, I now have a debt, meaning, less than zero.*

*8. Samia: The answer is less than zero because we had positive 34 and negative 48, so we subtracted them from each other, so the answer would be -14, which is less than zero.*

The automatic feedback provided by the platform allowed the teacher to attend that the correct answers are about 50% , after monitoring students' answers we see that the teacher used a question that explored the students' answers (Line 4). Muhammad, Jannah, and Samia told the teacher and the other students how they arrived at their answers. Muhammad used the number line, Jannah used everyday life, and Samia used the rules. We can see here that following one exploring question, three students participated in the discourse.

**Excerpt 3.** After asking a question via the platform of interactive presentation, the teacher displayed a selected student's answer and discussed it with the students.

*1. Teacher: (A student's answer is displayed on the computer screens.) What do you think of the student's answer?*

*2. Muhammad: It's wrong.*

*3. Teacher: Who agrees with Muhammad?*

*4. Samia: Me, my teacher.*

*5. Janna: Me, my teacher.*

*6. Teacher: So, why do you think the student's answer is wrong?*

*7. Ayman: Because negative 400 is greater than negative 300?*

*8. Teacher: Ahmad, what do you think of Ayman's answer? Is negative 400 greater than negative 300?*

*9. Ahmed: No, my teacher? Negative 400 is smaller because it's to the left of negative 300.*

In Excerpt 3, the automatic answers were filtered according to their correctness, which allowed the teacher to attend and monitor the wrong answer and to share with the other students three types of questions that might promote mathematical discourse. First, in Line 1 the teacher revoiced one answer, after that in line 3, we see the encouragement of interaction among the students, with questions that aim to explore how the students arrived at their answers. In Line 8, we see a question that aimed to incorporate students' background knowledge. Five students participated in the discourse in this excerpt.

In Excerpt 4, the teacher introduced a student's solution to the following question: "Solve without calculating the result of the exercise. Is it located to the right of the zero or to the left of the zero? Explain your response."

*1. Teacher: Wonderful Samia, who can tell me what can we conclude? An algebraic rule that we can deduce?*

*2. Layan: What is meant is that if its absolute value is greater, the answer sign will be the same.*

*3. Teacher: Good, you've told us a part of the rule.*

*4. Samia: We take the absolute value of the two numbers and the absolute value of the greater number by taking its sign and subtracting the smaller number from the larger one.*

In Excerpt 4, the carpet answer makes it easy to monitor which students answer the question correctly. The teacher's question aimed to value the student's idea. In this very short excerpt, two students participated in the discourse.

*Analysis based on narratives*

In this section of the analysis, we will present data that describe narratives about numbers presented by the teachers and the students in the experimental group and the control group. Data from the first lesson are presented in Table 5 and data from the second lesson are presented in Table 7. After presenting each table, we analyze one excerpt from the classroom discourse.

*First Lesson: Adding Together Two Numbers With The Same Sign*

*Table 4. Mathematical narratives presented by teachers and students during the first lesson*

| | **Number of teachers' narratives** | **Examples of teachers' narratives** | **Number of students' narratives** | **Examples of students' narratives** |
|---|---|---|---|---|
| Control group | 8 | The sum of two negative numbers is always a negative number.<br>The further we go to the left on the number line, the smaller the number is. | 10 | According to the rule of two positive numbers, and the two negative numbers are similar to the sign, we degrade the negative sign by the answer, and then we add the two absolute numbers.<br>The sum of two negative numbers is smaller than the additives. |
| Experimental group | 18 | The sum of two negative numbers is smaller than the additives.<br>The sum of two negative numbers is always a negative number. | 27 | The more leftward we go on the number line, the smaller the number.<br>The sum of two negative numbers gets smaller because the addition of two negative numbers. |

During the first lesson, the teacher and students in the experimental group presented more narratives that could be accepted or rejected than by the teacher and students in the control group (Table 4). The students in the experimental group presented 17 more narratives than the students in the control group. Similarly, the teacher in the experimental group presented 10 more narratives than the teacher in the control group.

An example from the experimental group's lesson on adding together two numbers that each have a negative sign is presented in Excerpt 5.

**Excerpt 5.** The teacher used the platform of interactive presentation to display a question.

لايجاد رصيد حساب اديب سجلت الموظفة
(-700) + (-500) =(+1200)
هل حساب الموظفة صحيح/ ام غير صحيح
اذا كان صحيح ..اشرح
اذا كان غير صحيح، ، اشرح وسجل التمرين الصحيح الذي يصف حساب اديب

*1. Ahmed: It is not correct because there is a mistake in the sign.*

*2. Teacher: So, what do you think? What is the answer?*

*3. Hasan: It's wrong, Teacher, because the sum of two negative numbers is a negative number, and here we have a positive number.*

*4. Teacher: Who agrees with Hasan?*

*5. Dina: I do.*

*6. Samia: Me, too. It should be negative 1200.*

*7. Ahmed: The sum must be smaller, not bigger, because we have two negative numbers.*

*8. Teacher: Listen to what Ahmad said, an important sentence. When the two additives are negative, is their sum smaller than each of the additives?*

*9. Samia: Yes, I can explain, because if we take it on the number line, the steps go to the left, and the further we go to the left, the smaller the number gets.*

*10. Teacher: Wonderful, well done. You saw how Samia linked Ahmed's claim to the number line? Wonderful. From the discussion, we can conclude the following about the sum of two negative numbers: The sum is less than zero, which is always a negative number. And the sum is smaller than each of the additives.*

*Table 5. Analysis of narratives presented by the teacher and students in Excerpt 5*

| **Narratives** | **Narrative presentation** | |
|---|---|---|
| | **Narratives presented by the teacher** | **Narratives presented by students** |
| The sum of two negative numbers is a negative number. | | A student's good interpretation of the question presented by the teacher (Line 3) |
| The sum of two negative numbers is smaller than each of the additives. | | Ahmed used this narrative to explain Samia's answer to the question. (Line 7) |
| The sum of two negative numbers is smaller than each of the additives. | The teacher restated what Ahmed said, in order to emphasize the importance of what he said and presented it again in the form of a question. (Line 8) | |
| The further to the left we go on the number line, the smaller the number gets. | | Samia's explanation of the teacher's question, "Is the sum of two negative numbers smaller than the additives?" She used the number line to interpret the claim. |
| The sum of two negative numbers:<br>A. The sum is less than zero (always a negative number).<br>B. The sum is smaller than each of the additives. | Neat summary of the students' conclusions (Line 10) | |

In that short excerpt, three narratives were presented by students and two narratives were presented by the teacher. The teacher recognized a question that many students have difficulty solving, according to the automatically filtered answers. The platform allows teachers to monitor and attend to problematic questions according to the percentage of student responses that are correct. The teacher can respond by offering his or her own narrative or by asking questions that might facilitate sequencing and connecting between students' responses. The students can then react with narratives.

*4.2.2. Second Lesson: Adding Together Two Numbers That Have Different Signs*

*Table 6. Mathematical Narratives Presented by the Teachers and the Students During the Second Lesson*

| | **Number of teachers' narratives** | **Examples of narratives presented by the teachers** | **Number of students' narratives** | **Examples of narratives presented by students** |
|---|---|---|---|---|
| Control group | 10 | We take the sign of the number whose absolute value is greater, meaning that we take the sign of negative 8 because its absolute value is greater.<br>The absolute value is the number's distance from zero and that dimension is always positive. | 11 | The absolute value parameter refers to when we take the number from negative to positive.<br>Counter numbers are the same distance from zero. |
| Experimental group | 22 | The absolute value is the number's distance from zero, so it is always a positive number or zero.<br>Initially the sign of the sum is the same as the sign of the number with larger absolute value, and then we make the difference between the number with the largest absolute value and the number with the smallest absolute value. | 35 | The distance between a number and zero, is the absolute value of that number.<br>We take the absolute value of the two numbers, we take the sign of the number whose absolute value is the greater, and subtracting the absolute values of the numbers. |

The teacher and the students in the experimental group mentioned more narratives than the teacher and students in the control group (Table 6). There was a big difference between the number of narratives mentioned by students in the experimental group and the number of narratives mentioned by students in the control group. The students were motivated to participate and put forward their ideas and we can see that the platform of interactive presentation encouraged their learning.

In this section of the analysis, we will deal with an excerpt from the lesson entitled "The sum of two numbers with different signs" (Excerpt 6). This excerpt shows a discourse about a student's solution to the question: "Solve without calculating the result of the exercise: Is it located to the right of zero or to the left of zero? Explain your response!" $((+43) + (-48))$

**Excerpt 6.** The teacher presented a student's solution to the question:

*1. Teacher: Dunia interpreted her answer as follows: "It is to the left of zero, positive 34 is greater than zero, but negative 48 is greater than positive 34, meaning the answer is less than zero." Can you explain your answer?*

*2. Dunia: Because 48 is greater than 34.*

*3. Teacher: What do you mean by greater?*

*4. Dunia: I mean 34 is greater than negative 48 in terms of value, but the regular number 48 is greater.*

*5. Teacher: What do you mean by regular number?*

*6. Dunia: I mean, I mean, the number of steps from zero, for the number 34 and the steps for negative 48.*

*7. Teacher: What you said the number of steps is the distance from the zero. What do you think?*

*8. Samia: The distance of the number from zero, we learned that that's the absolute value of the number.*

*9. Teacher: Wonderful, fantastic. The distance of the number from zero is the definition of the absolute value. Who can tell me what can we conclude? An algebraic rule that can we deduce?*

*10. Layan: What is meant is to take the sign of a number whose absolute value is greater.*

*11. Teacher: You mentioned a part of the rule.*

*12. Samia: We take the absolute values of the two numbers and the absolute value of the greater number by taking its sign, and subtracting the smaller number from the larger number.*

*13 .Teacher: Wonderful. The distance from zero is the definition of the absolute value, but I want to focus on using more mathematics. The sign of the sum is the same as the sign of the larger number with its absolute value, as the sum is the difference between the number with the largest absolute value and the number with the smallest absolute value.*

*Table 7. Analysis of the narratives mentioned by the teacher and students in Excerpt 6*

| Narratives | Narrative presentation | |
|---|---|---|
| | Narratives presented by the teacher | Narratives presented by students |
| Positive 34 is greater than zero, but negative 48 is greater than positive 34. | | Interpreting Dunia's answer: The sum is less than zero. (Line 1) |
| The number of steps from zero for 34, and the number of steps from zero for negative 48. | | The teacher asked the student, "What do you mean by regular numbers?" (Line 5) |
| The distance from zero is the absolute value of the number. | | The teacher asked what is meant by the number of steps from zero. Samia replied, "The distance of the number from zero, the absolute value of the number." (Lines 7 and 8) |
| Take the sign of the number whose absolute value is greater. | | When the teacher asked to deduce the algebraic rule, Layan answered. (Line 10) |
| We take the absolute value of the two numbers, and the absolute value of the greater number by taking its sign, and subtracting the smaller number from the larger number. | | Student's deduction of the algebraic rule (Line 12) |
| The sign of the sum of the sum is identical to the sign of the larger number with its absolute value, as the sum is the difference between the number with the largest absolute value and the number with the smallest absolute value. | Concise restatement of students' formulation of the algebraic rule (Line 13) | |

In Excerpt 6, five narratives were presented by the students and only one narrative was presented by the teacher (Table 7). This is similar to what we saw in the excerpt analyzed in Table 5. We see in these episodes that, in the experimental group, the students presented more narratives than the teacher did.

## Discussion

In the current study, we investigate whether:

1. Using an interactive presentation (IP) platform could affect the amount of usage of the questions related to the fourth and fifth practices of *sequencing* and *connecting* students' responses.

2. Using the platform of interactive presentation (IP) affects the amount of students' participation with *narratives* during the mathematical discourse.

In this study, we revealed that the platform of interactive presentation might allow the teacher to use three of the five strategies, namely, clarifying and sharing learning intentions, the architecture of effective discussions, and providing feedback that helps students to progress in their learning (Black & Wiliam, 2009). When articulating among the students the teacher has fewer opportunities to monitor all the student responses, the interactive-presentation platform automatically filtered feedback gives more opportunities to *easily attend* and *monitor* one student contribution and the whole class contributions after the teachers' analysis of the students responses (Olsher & Abu Raya, 2019) they respond by either question which might promote productive mathematical discourse by making *sequencing* or *connecting* students' responses or narratives presented him/her flowed by narratives presented by students (Jacobs et al., 2010; Stein et al., 2008; White, 2003)

In this study, there is a tendency that using the platform of interactive presentation (IP) might allow the teacher to ask nearly three times more questions that sequencing and connecting students' responses compared with the teacher who was not using the platform of interactive presentation (Stein et al., 2008; White, 2003), All of the different types of questions which connect students' responses (i.e., valuing students' ideas, exploring students' answer, incorporating students' background knowledge, and encouraging student-to-student communication) were asked more often in the experimental group than in the control group (see Tables 2 and 3). Comparing to the situation before IP intervention in which the number of questions from all types was almost the same in the experimental group and the control group (see Table 1.b). Excerpts 1 and 2 reinforce this conclusion. In those excerpts, the teacher used more than one type of discourse-promoting question and at least three students participated in the discourse. By using the platform of interactive presentation, the teacher was able to apply the fourth strategy for promoting discourse as coined by Wagganer (2015), that is, the comparison between the explanation and the justification process, as well as the fifth strategy of giving examples of the positions.

The use of the platform of interactive-presentation technology can advance the teaching of mathematical content (Drijvers, 2013; Drijvers et al., 2009; Martin, 2008). Through the program, the students have the opportunity to participate in activities that encourage them to think. Afterward, the teacher can make decisions about which of the responses submitted by the students he or she would like to present to the other students (Burns & Polman, 2006; McKay & Ravenna, 2016). The platform of interactive presentation allows the teacher to see the different answers and to make formative assessments when asking suitable questions, in order to motivate other students to participate in the mathematical discourse (Aldon et al., 2017).

Regarding the second question, we saw that, in the lessons involving the use of this platform, the students tend to participate in the mathematical discourse and were agents of their own learning, this is in the same direction as the results of Cooper et al. (2020). We saw that more narratives were presented by the teacher and students in the platform of the interactive-presentation group than by the teacher and students in the control group. In the first lesson, the students in the experimental group presented 27 narratives; whereas the students in the control group presented only 10. In the second lesson, the students in the experimental group presented 35 narratives; whereas the students in the control group presented only 11 (see Tables 4 and 6). Clearly, the students presented more narratives when the teacher used the platform of interactive-presentation technology. This was also seen in our analysis of two short excerpts from our transcripts (see Tables 5 and 7). We saw that the use of the platform of interactive-presentation technology allowed the teacher to use the strategies of activating students as learning resources for each other and activating students as the owners of their own learning (Black & Wiliam, 2009; Yerushalmy & Olsher, 2020). These activities might facilitate the discourse in the classroom, the IP might allow effective interaction in the learning sequences (Abdu et al., 2021).

Our results underscore the argument that a platform of interactive presentation might facilitate mathematical conversations, with students participating by guessing, identifying, and explaining. This might help them to expand, enrich, and develop their understanding of mathematical concepts and aid their subsequent learning by providing opportunities for them to use mathematical narratives and train themselves (Brown, 2010; Chapin et al., 2003; Davis, 1997; Kersaint, 2015). These results are in the same direction as other studies which emphasized the advantage of using digital automatic platforms in the teaching processes (Haj-Yahya & Olsher, 2022; Olsher et al., 2016; Popper & Yerushalmy, 2021).

## Conclusion

To conclude, we revealed that the use of an interactive presentation platform could increase the amount of usage of questions related to the fourth and fifth practices of sequencing and connecting students' responses. Furthermore, the use of an interactive presentation platform could also enhance students' participation with narratives during mathematical discourse.

## Recommendations

Our recommendations for instruction in the area of teacher training are as follows. During their training, teachers should be exposed to the specific features of this technology that were revealed in this study. This exposure might help mathematics teachers to diagnose and think through students' difficulties, perform better as teachers, and improve student achievement.

Further studies should also involve a larger and more diverse population. In the current study, we chose to examine math classes characterized by heterogonous achievement levels. The question arises as to whether similar results would be observed among more homogenous math classes. We recommend that future studies also include high-school students, as well as populations from different sectors in society and other parts of the world. Future studies might also explore the impact of using another platform that includes other pedagogical aspects, such as examples that vary not only in their degree of correctness.

## Limitations

The main limitation of this study is that two different teachers taught the control group and the experimental groups, which may have increased the variability of the results and could limit the generalizability of our findings. Although the lessons taught, were the same ones outlined in the lesson plans, it still difficult to state that the data are completely comparable. Further studies might attempt to confirm our findings in a system in which both the control and the experimental group experience the same research tools and the same teacher.

### Data Availability

The data that support the findings of this study are available on request from the corresponding author.

### Authorship Contribution Statement

Haj-Yahya: reviewing, supervision conceptualization, design, critical revision of manuscript, editing. Aegbaria: data acquisition, analysis, interpretation, writing.

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## Appendix

4.1.1 First Lesson (90 minutes):

Adding two positive numbers, adding two negative numbers, where the lesson contains methods for calculating the addition of numbers directed to two similarly signed numbers, questions that raise the class discourse. The aim of the lesson, to illustrate the addition process in three ways: the axis of numbers, the daily life and the algebraic method.

Lesson Two (90 minutes):

adding two numbers with different signs, the lesson contains brainstorming questions, explanation and explanation questions, motivational questions, writing problems from daily life that embody a given addition exercise, the goal of the lesson to solve questions in three ways: the axis of numbers, daily life and the algebraic method (knowing the relationship of absolute value to the Addition of vector numbers), the sum of two opposing numbers.

4.1.3 Lesson Three (45 minutes):

Exercises that contain more than one additive, the goal of the lesson: to enable the student to solve exercises of adding more than one, to identify methods and laws for solving exercises. The lesson contains critical questions in which the student's analytical methods have grown.