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On Pre-Service Teachers' Content Knowledge of School Calculus: An Exploratory Study

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Abstract: This paper reports an exploratory study on the pre-service teachers' content knowledge on school calculus. A calculus instrument assessing the pre-service teachers' iconic thinking, algorithmic thinking and formal thinking related to various concepts in school calculus was administered to a group of pre-service mathematics teachers. Their performance on five of the items is reported in this paper. Other than their good performance in the iconic recognition of stationary points, their recognition on points of inflexion, differentiability and notion of minimum points was relatively poor. In addition, they appeared to lack the algorithmic flexibility in testing the nature of stationary points and the formal thinking about definition of an extremum point. The implications of the findings are discussed.

Keywords: *Algorithmic thinking; formal teaching; iconic thinking; pre-service teachers; school calculus knowledge.*

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Introduction

The importance of teacher quality in an education system cannot be underestimated. According to the McKinsey Report (Mourshed et al., 2010) on School Quality, "*the quality of an education system cannot exceed the quality of its teachers.*" (p. 7). One of the important aspects of teacher quality is the teacher's mastery of content knowledge. Studies have shown that content knowledge is highly predictive of teachers' mathematical pedagogical knowledge, inclusive of the ability to plan instructional support (e.g., Norton, 2018). Mastery of content knowledge enables teachers to appreciate the connections across various domains of mathematics, so that they are able to develop their students' problem-solving ability (Yan et al., 2022).

Shulman (1985) identified content knowledge as one of the three aspects of teachers' professional knowledge. This content knowledge that teachers need to know describes their understanding of the teaching subject, which is "a deep understanding of the domain itself" (Shulman, 1986, 1987).

We are not debating on what exactly constitutes content knowledge that teachers need to know. The fine-grained detail has been discussed in great length by various researchers (e.g., Ball et al., 2008; Hill et al., 2004; Krauss et al., 2008; Stacey, 2008). In this paper, we focus on pre-service teachers' content knowledge of school mathematics – the knowledge that any beginning calculus teacher should know, and not on the specialized content knowledge of a schoolteacher. In particular, we focus on school calculus knowledge.

There have been several studies on teachers' content knowledge of school mathematics within various national contexts (e.g., Linsell & Anakin, 2012; Livy & Vale, 2011; Olanoff et al., 2014; Stohlmann et al., 2012; Toh, 2017; Toh et al., 2007; Venkat & Spaull, 2015), and in international comparative studies (e.g., Toh et al., 2013). However, these studies do not focus on calculus content knowledge. Most of the tasks on assessing teachers' content knowledge, including the tasks on calculus if any, are computational in nature and assess procedural knowledge in calculus. The study reported in this paper

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examines pre-service teachers' knowledge of school calculus concepts of stationary points, inflexion points, maximum and minimum points.

The performance of a group of pre-service mathematics teachers in a calculus instrument after their admission into the local teacher education institute for the pre-service teacher education programme and prior to embarking on their first undergraduate mathematics courses is reported in this paper. The result of this study threw light on their entry knowledge of school calculus.

This group of pre-service teachers (PSTs) had attended a common mathematics curriculum at the secondary and the pre-university levels (the mainstream schools in Singapore covered a common mathematics curriculum) and sat the same high-stake national examinations at the end of the secondary (GCE O-Level examination) and pre-university education (GCE A-Level examination). All the PSTs had read calculus at the secondary and pre-university levels. Hence, their performance in the calculus instrument can be seen as a measure of their knowledge of calculus up to the pre-university calculus, prior to their admission to the university education. Thus, this study, in adding to our understanding of the calculus content knowledge of pre-service teachers at the university entry level, provides clues for a review of the secondary and pre-university calculus curriculum content and for university professors in designing their beginning undergraduate calculus courses.

The PSTs selected for this study were among the best of their peers based on their pre-university academic performance and the stringent criteria to be eligible for one of the most prestigious teaching scholarship awards in the country. Thus, their performance might not be representative but a "best-case scenario" of all the beginning undergraduate mathematics students in Singapore; their inability to solve an item could be taken as an indication of what the general beginning undergraduates are likely unable to solve.

Calculus is an important strand in the upper secondary and pre-university school mathematics curriculum in many countries as it is a vital pre-requisite for STEM Education. Many studies in various national contexts have shown that calculus is difficult for both students and teachers (e.g., Amit & Vinner, 1990; Ng & Toh, 2008). A small-scale study on a small group of practicing teachers, all of whom were experienced high school teachers, showed that their knowledge of school calculus is mainly restricted to procedural knowledge without conceptual understanding (Toh, 2009). In the recent study by Toh et al. (2021), it was revealed through two related calculus tasks on limits and differentiation that pre-service teachers' knowledge on calculus is largely procedural; hardly any of them identified the connection between two related tasks of derivative and limits.

Supported by education research literature, we as teacher educators believe that conceptual knowledge is extremely important for teachers. Only with strong conceptual knowledge are teachers able to understand the algorithms used in completing the various mathematical tasks and devise expedient means to engage their students (Cho & Nagle, 2017). It is a common knowledge that how a subject is being taught in schools is heavily dependent on the teachers' mastery of the subject knowledge (e.g., Thomson et al., 1992). Only when we understand the specific difficulties faced by teachers can we really address any imperfection of calculus teaching in schools.

Literature Review

One of the attributes of the school calculus in the Singapore school mathematics curriculum is a heavy emphasis on developing students' procedural knowledge over conceptual knowledge (Toh, 2021). Procedural knowledge refers to knowledge about rules, algorithms, and procedures that are used to perform mathematical tasks. Conceptual knowledge includes not only knowledge of pieces of information, but also the relationship between the various pieces (Hiebert & Lefevre, 1986). However, researchers such as Rittle-Johnson et al. (2001) asserted that conceptual and procedural knowledge should not be viewed as mutually exclusive. The increase in one type of knowledge will foster the development of the other in the category. We will next turn our discussion to students' thinking about calculus.

Aydin and Ubuz (2015) identified six aspects of mathematical thinking: enactive, iconic, algorithmic, algebraic, formal, and axiomatic thinking. These six aspects of mathematical thinking are a natural corollary of the three-world framework (perception, operation and reason) built by Tall (2004), who based his framework on Bruner (1971) and Hughes-Hallet (1991). We briefly discuss three out of the six mathematical thinking discussed by Aydin and Ubuz (2015): iconic, algorithmic, and formal thinking in this section.

Iconic thinking refers to the visualization used by individuals when making use of images, diagrams and graphs to reflect and interpret so as to represent and communicate information (Arcavi, 2003). In particular, iconic thinking includes extracting and interpreting information from graphs (e.g., Meletiou-Mavrotheris & Lee, 2010). Thus, iconic thinking is an important part of calculus education, in which learners begin to appreciate most concepts by its informal graphical interpretation (Toh, 2009).

Algorithmic thinking refers to the thinking involved in selecting the correct algorithms to solve a given problem (Martin, 2000). This is also referred to as the usual procedural knowledge. School calculus curriculum, usually referred to as an *informal calculus*, tends to focus more on procedural understanding and algorithmic thinking (Toh, 2021).

Formal thinking involves constructing meaning from definitions, facts and symbols (Tall, 2004). It focuses on the factual information that underlies the fundamental mathematical concepts.

The above three modes of mathematical thinking were used as our theoretical basis in developing our items in relation to understanding students' image or iconic thinking of various calculus concepts, their ability to link their procedural knowledge to calculus concepts, and the formal definition of calculus concepts. These are discussed in detail below.

Calculus items used in this study

For the convenience of the readers, the items that we had constructed for the study are represented in Appendix A in order to illustrate how the items were constructed based on the above three types of thinking.

Item 1 examines the students' iconic thinking about several closely related calculus concepts in school calculus curriculum: maximum / minimum points and stationary points; differentiable points on a function. The options (A), (B) and (C) examine the solvers' ability to move between algorithmic thinking to iconic thinking, that is, the ability to translate their concept definitions to visual images of the calculus concepts. The content knowledge of this has been discussed at great length in Toh (2008). The option (D) aims to examine the students' iconic thinking of stationary point and recognition of a non-differentiable point. Students' anticipated notion of the derivative at cusps can fall under the category of the errors discussed in Tsamir et al. (2006): (1) an over-generalization of the procedural knowledge, based on the piecewise defined function of the absolute value function; (2) an over-generalization to that absolute value function is non-differentiable everywhere.

Making sense of the concepts of stationary points and points of inflexion through making a connection between their algorithmic thinking and their iconic thinking is studied through items 2 and 3. These two items require the solvers' visual inspection of the notions of stationary points and inflexion points, the concepts of which were usually presented algorithmically in textbooks. Although calculus concepts such as the first and second derivatives can be represented through three symbolic representations (symbolic, graphic and numeric), students might not recognize or use the relationship between them (Asiala et al., 1997; Aspinwell & Miller, 1997; Schwarz & Hershkowitz, 2001). We noted that the local school curriculum emphasizes much of algorithmic thinking in locating these points (equating the first derivative to zero in order to find the stationary points and equating the second derivative to zero to find points of inflexion), without an elaboration on recognizing these concepts visually.

The concept of inflexion points is first introduced at the secondary calculus curriculum without the notion of concavity. The rationale of this approach is to use inflexion point as a classification of stationary points which are neither minimum nor maximum points (Toh, 2021). This over-simplistic procedural classification could point to plausible misconception about the concept of inflexion points among university students (Tsamir & Ovodenko, 2013). These two items examine the proficiency of the sovlers in aligning their iconic and algorithmic thinking about their knowledge of the first and second derivatives.

Item 4 involves formal thinking about a maximum point. Since many mathematical concepts, especially the advanced concepts in calculus, are not formally defined in the school mathematics curriculum, students are usually taught to recognize them through informal experience (Tall & Vinner, 1981). Thus, whether students are eventually able to recognize the formal definition from their usual procedural approach of finding a maximum point, that is, move between formal and algorithmic thinking, remains implicit. An algorithmic approach to finding a maximum (or minimum) point involves finding the coordinates of the point for which the first derivative is zero, and checking that the second derivative is negative (positive respectively), or the first n derivatives to vanish and has a negative value (positive respectively) for the first (n+1)-th derivative, and where n is even. Our collective classroom experience shows that being obsessed with algorithmic procedures related to school tests for the various calculus concepts at the schools might have distracted students from the more fundamental formal definition of the related calculus concepts.

Item 5 involves algorithmic thinking on determining the nature of stationary point. It is in the syllabus document that students at the secondary level should have been introduced to the use of both the first and the second derivative tests to determine the nature of stationary points. It is thus expected that students need to have the ability to make their own judgement about the most appropriate or efficient procedure for a particular instance (e.g., National Research Council, 2012; Star, 2005). In this item, the solvers will need to make their judgement on the suitability of the first derivative test over the second derivative test, where the latter fails for this item. The purpose of this item is to examine the procedural flexibility of the students. Citing Maciejewski and Star (2016), we need not be apologetic that some degree of algorithmic instruction is required for advanced mathematics such as calculus. A focus on procedural knowledge can be seen as an opportunity to impart flexibility and a richer view of the roles algorithms play in more advanced mathematics.

With the use of the above items, this study aims to answer the research questions (RQs):

RQ1. What are the pre-service teachers' images of the calculus concepts: maximum, minimum, stationary points and differentiability of a function at a point?

RQ2. How do the pre-service teachers identify a maximum point by its formal definition as distinguished from the algorithms to determine its nature as a maximum point?

RQ3. Are the pre-service teachers able to select or switch to an appropriate procedure when determining the nature of a stationary point?

Methodology

The Calculus Instrument

A calculus survey instrument was constructed for our exploratory study of pre-service teachers' (PST) content knowledge of calculus. We decided on the format of multiple-choice questions for the calculus survey instrument. The use of multiple-choice items has its advantage in that it is convenient for online administration of the survey. The researchers had initially considered using open-ended tasks, but were cognizant of the lack of domain coverage in using open-ended tasks (Parke et al., 2006). Pertaining to multiple-choice format, we were also mindful of the possibility of the PSTs attempting multiple-choice items by using mere guessing, which would reduce the validity of the findings. Thus, we deliberated on the choices of distractors as visibly suitable choices of the answers.

The survey consisted of 16 items on various aspects of calculus. In this paper, we select a cluster of items (Table 1) which examine the concepts related to (i) maximum, minimum and turning points; and (ii) stationary points and points of inflexion. The matching of the mathematics concepts and the three forms of thinking that are being studied in the above items are summarized in Table 1 below.

Table 1. Matching of the questions and the underlying thinking

	Iconic thinking	Algorithmic thinking	Formal thinking
Maximum / Minimum / Turning point	Item 1		Item 4
Stationary and inflexion points	Item 1, Item 2, Item 3	Item 5, Item 2, Item 3	Item 5

As shown in Table 1, the concepts identified in this study were categorized under two main groups: (1) maximum, minimum and turning points; and (2) stationary points and inflexion points. Items 1 and 4 were designed to study the PST's iconic and formal thinking about the first category of concepts. Items 1, 2, 3 and 5 were designed to study their iconic, algorithmic and formal thinking about the second category. In other words, the PSTs' "concept image" and formal thinking (or definition) of both categories of concepts, and, in addition, the algorithmic thinking related to the second category of concepts were investigated by the items (Appendix A). It is particularly interesting for the algorithmic thinking associated with the second category of concepts since these algorithms associated to locating the stationary and inflexion points are only necessary but not sufficient.

Participants and Data Collection

The participants for this study were from one entire cohort of pre-service teachers (PSTs) newly admitted to one local undergraduate programme prior to their reading of the first undergraduate mathematics programme (calculus and linear algebra). The cohort consisted of 22 candidates, out of which 20 of them participated in this study willingly upon invitation and according to the University's approved ethics guideline. In this paper, in order to maintain anonymity, the 20 PSTs are labelled T1 to T20. The PSTs enrolled into this programme were selected through a series of very stringent criteria. Thus, this group of students was not representative of the entire cohort of undergraduate students admitted to the various universities during the same year of admission, but they were among the best of the students among their peers.

The calculus survey was administered to the participating PSTs through an online version in which they were required to answer them within one laboratory session (only five of the items which are relevant to the study described in this paper are reported here). Graphing and scientific calculators were allowed for their completion of the calculus survey. All the participants completed the items within an hour. To minimize peer influence within the lab, the ordering of all the items, including the order of the multiple choices within each item, in the survey were randomized for all the participating PSTs. In addition to asking the PSTs to select the most appropriate answer for each item, the participants were encouraged but not mandated to give a reason for their choice of answer for each item. Their open-ended answers and comments would provide the researchers a better understanding of their choices of answers.

Data analysis

The PSTs' responses were collated using a Microsoft excel spreadsheet. As described above, all the multiple choices for each item presented to the PSTs were randomized (so that each PST encountered different orderings of the choices for each item). The PSTs' answers for each item were labelled in the excel spreadsheet according to the items in the researchers' master copy (Appendix A). To ensure the validity of the PSTs' responses, all their responses were checked against their open comments for the corresponding items (if any) to check for consistency.

Findings / Results

The result of the PSTs' responses to the above items is tabulated in Table 2. The correct answer of each item is marked with an asterisk below. The remark column describes the objective of the item.

Table 2. Students' responses to the items in the calculus survey

Item	(A)	(B)	(C)	(D)	(E)	Remark
1	5* (25%)	1 (5%)	2 (10%)	6 (30%)	6 (30%)	Visual inspection of minimum point
2	0 (0%)	0 (0%)	1 (5%)	19* (95%)	0 (0%)	Visual inspection of stationary points
3	14 (70%)	0 (0%)	0* (0%)	6 (30%)	0 (0%)	Visual inspection of inflexion points
4	5* (25%)	3 (15%)	7 (35%)	3 (15%)	2 (10%)	Formal definition of maximum point
5	0 (0%)	7* (35%)	6 (30%)	7 (35%)	0 (0%)	Test nature of stationary point

Pre-service teachers' images of stationary, maximum, minimum, inflexion and non-differentiable points

The answers to RQ1 on the pre-service teachers' images of stationary, maximum, minimum and inflexion points were elicited from their responses to items 1, 2 and 3 (Table 2). These concepts are easily confused among school students (Toh, 2021), and even among practicing experienced mathematics teachers (Toh, 2009).

For item 1, which studies the PSTs' visual recognition of a non-stationary minimum point (hence not a stationary point), only 25% of the PSTs recognized the origin (0,0) as a minimum point. In addition, as shown in Table 2, 30% of the participants (who chose the option (D)) mistook the cusp at the origin as having zero gradient, an indication that they had not encountered functions with non-differentiable points, or were not able to visually detect cusps as non-differentiable points. Another 30% of the participants selected the option (E) for this item, indicating that they took the first four statements as false. In particular, these participants who chose option (E) failed to recognize the origin as the minimum point. In this latter group, four of the six PSTs provided their reasons in the open response section which are presented below. The erroneous parts of their responses are italicized for the ease of reference in the discussion that follows.

T1. If it is a stationary point that is a minimum point, dy/dx will be zero and it will be a turning point. Statements 1,2,4 contradict statement 3. Thus, I feel that none of the above statements is true.

T2. The graph of y=|x| is a combination of both the y=-x line (when x<0) and y=x line (when x>=0). As such, *the gradient of the graph [at x=0] is either 1 or -1 and therefore not 0*. It is not a stationary point, turning point nor minimum point as the dy/dx of the point (0,0) is either 1 or -1. It is not 0.

T11. I think the gradient can be interpreted from both graph [i.e., to the left and to the right of *x*=0] +1 or -1??

T18. (0,0) is called the vertex of the graph.

The PST T1 mistook that a stationary point must be a turning point, through a series of formal reasoning (that of relationship between mathematical properties). In other words, the iconic thinking involving a maximum or minimum point was not complete in T1, whose response above further showed that the relation between stationary point, turning point and points at which gradient equals zero was unclear. In view of the unclear relationship of these concepts and a lack of a complete iconic thinking of a minimum point, the PST failed to recognize the minimum point which is not a stationary point. On the other hand, T18 labelled the non-differentiable point (0,0) as vertex, a term most likely borrowed from Euclidean geometry as in the vertex of a polygon. T18 attempted to fit a form of formal thinking (definition of a mathematical property) into a new context without relating it to the calculus concepts, although unsuccessful in this case.

Evidently most of the PSTs did not recognize the existence of non-differentiable point (0,0). While 30% (who chose option (D)) asserted the gradient at (0,0) to be zero, others (who chose option (E)) contemplated on the value of gradient by examining the graph from both sides of (0,0), a behaviour that resembles an undergraduate calculus student's examination of one-sided limits. It was interesting to observe that both PSTs T2 and T11 attempted to apply their iconic reasoning on derivative as gradient at the point (0,0) by examining from the left and the right sides of the function, and concluded that the gradient at (0,0) is either +1 or -1. Underlying T2 and T11's attempt to decide the value of the gradient, they believed that a value of the gradient exists although they admitted they were unable to determine its value. They did not recognize that this is a non-differentiable point.

For Item 2, the PSTs have an overwhelming high percentage of correct response (95%). The PSTs (including the PST T10 who gave the incorrect response) were able to comment that the stationary points have gradient zero. It appeared that T10's incorrect response was a slip, as judged from his written comment to this item below.

T10: Stationary points are points whereby the rate of change y/rate of change x = gradient of tangent at that point = 0 (Tangent is horizontal line, parallel to x-axis).

For Item 3, none of the PSTs provided the correct response. In fact, 70% of the PSTs indicated that there were no points of inflexion on the given graph (option (A)) and the remaining 30% responded there were three points of inflexion. A review of the comments of the PSTs that reflected their understanding of points of inflexion shows that their (mis)understanding fell under three categories: (1) unable to distinguish between maximum / minimum point and point of inflexion (exemplified by T2 and T10); (2) recognize that the second derivative of the function vanishes at points of inflexion and could not identify it from the graph (illustrated by T7); and (3) identify the procedure for checking for the nature of stationary point for a point of inflexion but did not articulate the essence of a point of inflexion (shown by T8 and T13). In other words, categories (2) and (3) involve the use of partial algorithmic knowledge about a point of inflexion but not being able to articulate the essence of a point of inflexion in relation to the change of concavity of the graph.

T2: There are 2 min and 1 max points on the graph.

T10: There are 3 turning points so 3 inflexion points.

T7: Point of inflexion is a point where the second derivative equals 0, but there are no points on this graph reflecting this.

T8: A point of inflexion f(x) is either both increasing or both decreasing before and after the *stationary* [emphasis added] point.

T13: Points of inflexion are points where the graph will continue increasing or decreasing in the same direction.

There was no mention of the notion of concavity related to an inflexion point in any of the PST's response.

Pre-service teachers' operation of formal thinking about maximum point

The options for Item 4 were constructed to distinguish the PSTs' recognition of the formal definition of a maximum point, as contrast to the usual calculus procedural approach to determine a maximum point. In other words, this item studies the PSTs' formal thinking as distinguished from algorithmic thinking about the concept of a maximum point. Five (or 25%) of the PSTs selected the correct option for this item. Based on the open comments of item 4 provided by the PSTs who did not give the correct answer for this item, seven of the PSTs (35%) agreed that the description in the question could be a maximum point but were not certain about it as the function was not explicitly given. The comments of three of the PSTs, T6, T12 and T16, are produced below:

T6: Might be a bit tricky

T12: Need to differentiate

T16: Cannot solve [i.e., determine whether it is a maximum point]

Judging by the comments of T12 and T16, it was clear that the PSTs did not recognize the formal definition of a maximum point and considered it necessary to use the algorithm to determine the nature of the maximum point. The other three (or 15%) PSTs who chose the option (B) appeared to believe that they could not conclude for sure that the description fits the definition of a maximum point, but they need to use the first or the second derivative test to determine the nature of the point. The PST T11 commented that "Not sure how to do it. It seems like a maximum point to me." Collectively, the two groups of PSTs were not able to recognize the definition of a maximum point based on a generic function and in the absence of a specific function. For the five PSTs who chose the correct answer, T5, T7 and T19 commented on this item.

T5: The concept of a line of symmetry [?]

T7: If for all values of x, f(x) is always lesser than f(a), this means that (a, f(a)) is the highest point / max point.

T19: Since all f(x) other than a is smaller than f(a), it is the maximum turning* point of the graph.

It appeared that T5 had formed an incorrect concept image pertaining to a maximum point, even though option (A) was selected. The concept of a maximum point was associated with the existence of a line of symmetry passing through that point. T7 and T19 exhibited correct images of a maximum point based on its formal definition.

Pre-service teachers' switch between algorithmic procedures

Item 5 was constructed to study the PSTs' 'flexibility' (Maciejewski & Star, 2016) in selecting an appropriate procedure to perform the task of determining the nature of a stationary point using an appropriate method. The secondary and pre-university calculus curricula pointed out both the first and the second derivative tests to determine the nature of stationary points. Item 5 is one in which the second derivative test fails, and one needs to switch to the first derivative test (by examining the sign of the first derivative on either side of the stationary point). Anecdotal evidence from Singapore classrooms shows that students prefer the use of the second derivative test compared to the first derivative test, as the latter involves more analytic reasoning while the former is more procedural, as it involves computing the second derivative function followed by checking the sign after substitution. The second derivative test fails if its value equals zero.

In examining the PSTs' responses, all of them fall in the category of: (1) correctly recognizing the stationary point as a maximum point (35% or 7 of the PSTs selected option B); (2) erroneously identifying the stationary point as a point of inflexion (30% or 6 of the PSTs selected option C); and (3) asserting the lack of information to determine the nature of the stationary point (35% or 7 of the PSTs selected option D), probably in recognition that the second derivative vanishes.

In reading the open comment of the students who erroneously recognized the stationary point as a point of inflexion, the reasons can be grouped into two: the first group of PSTs (T2, T5 and T20) equated a point at which the first and second derivative equal zero as point of inflexion. In other words, they did not recognize that the vanishing of the first two derivatives is a necessary but not a sufficient condition for a stationary point of inflexion. Their comments are produced below.

T2: It is difficult to visualize the shape of the graph without plotting it. However, by substituting x=1 into dy/dx, we get dy/dx=0 because (x-1) is present. Calculating d2y/dx2, it still gives us 0 because (x-1) is still present, thus it is a stationary pt of inflexion.

T5: Since the second degree differential [i.e., the second derivative] still has the (x-1)^6 within the expression, it will give most likely give a point of inflexion at the point specified. [In other words, the second derivative will still give a value of zero]

T20: dy/dx=0, so when x=1, it is a stationary point. But differentiating dy/dx [once more] gives 0, so it is neither a maximum nor a minimum point.

The second group of students applied the first derivative test but somehow did not correctly match the result of the first derivative test with the consequence. Their comments are shown below.

T11: Sub[stitute] in values 0 and 2 to get gradient of graph before and after the stationary point (when x=1). [In other words, the PST used the first derivative test to check the sign of the gradient on either side of the stationary point but came to an incorrect result.]

T15: At x=0 dy/dx is 0, at x=0- [i.e., to the left of $x = 0$], dy/dx is negative, at x=0+ [i.e., to the right of x=0], dy/dx is positive thus it is a stationary point of inflexion.

T18: When x=1, dy/dx = 0. This shows that the gradient of the graph at x=1 is 0. when x=0.9, dy/dx is negative and when x=1.1, dy/dx is positive. Therefore at x=1, it is a stationary point of inflexion.

The error of this latter group of PSTs is a clear indication of them memorizing the first derivative test procedurally without associating the pictorial interpretation of the change of gradient with the nature of the stationary points. Consequently, such memory error is highly probable when no meaningful connection is being made.

In collating the open responses of the PSTs who chose option (D) as the answer for this item, it was interesting to recognize that this group of students neither used the first nor second derivative test to determine the nature of the stationary points.

T1: When x is 1, dy/dx is zero. Thus, it is a stationary point.

T6: [I] have to think a bit more to deduce the nature of the stationary point [but have not thought of how to do it].

T7: When x=1. dy/dx=0, which could indicate that it is either a min or max point. second derivative test required [but have not used it to check].

T10: When x is 1, the dy/dx is zero so it a stationary point but we do not know the nature of this stationary point.

T17: dy/dx = 0 shows us that it is definitely a stationary point where the gradient of the graph at the point is 0, however, it does not tell the nature [i.e., the PST did not use any test for the nature of stationary point].

The open responses of the PSTs who gave the correct response to this item are tabulated below. Only four out of the seven students provided the responses. It was clear that all of them applied the first derivative test to arrive at the correct conclusion.

T4: Using graphical method used in secondary school x=1 dy/dx is 0 x=1.1 dy/dx is =ve x=0.9 dy/dx is -ve [i.e., using the first derivative test]

T14: x= 1, gradient is zero. ok so its a stationary point. then, sub in x = 1.1, gradient = positive. sub x= 0.9, gradient is negative. so, gradient changes from negative to positive (left to right across x-axis). visually, it is a min, point.

T16: dy/dx = 0. But by subbing in 1- and 1+ into my equation of dy/dx. I deduce a minimum point.

T19: For all x, (1+x^10)^(1/2) is positive. For x < 1, (x-1)^7 is negative and for x > 1, (x-1)^7 is positive. Hence, when x < 1, dy/dx is negative and when x > 1, dy/dx is positive. This suggests a minimum turning point where the graph will have an upward concavity [it appeared that the PST had used the term concavity linking to the sign of the first derivative incorrectly].

The PSTs' responses to this item turned out to be a surprise to the researchers. The PSTs who used the first derivative test correctly gave the correct response (option (B)). The PSTs who applied the second derivative test and observed that the second derivative equals zero concluded that it was a stationary point of inflexion (T2, T5, T20); and the other students who selected (D) did not perform further calculation after realizing that the second derivative vanishes, most probably at a loss of procedure or an inability to switch to another procedure (the first derivative test). Thus, it appears that the second derivative test being inconclusive when the second derivative vanishes and the need to switch to the first derivative test did not seem to get through to the PSTs. The knowledge of both the first and second derivative tests does not seem to result in the PSTs' procedural flexibility and fluency.

Discussion

The PSTs' belief found in this study that the values of the derivative, represented graphically as the gradient of the function, exist for all functions and at all points (i.e., given any function, an expression for dy/dx can be determined) could be attributed to how calculus is taught in the secondary and pre-university calculus curricula (Ahuja et al., 1998; Toh, 2021). After several decades since the study by Ahuja et al. (1998), school calculus still pays much emphasis on procedural knowledge (or techniques), and in particular, differentiation techniques (Toh, 2021). With this almost exclusive emphasis on techniques of differentiation, students in the calculus section are mainly exposed to functions which are differentiable everywhere, without much cognizance of functions with non-differentiable points (e.g., the absolute value function is seldom discussed in calculus sections of the school syllabuses). Coupled with the absence of the formal definition of derivative in the school curriculum, students could undoubtedly be misled to believe that all functions are differentiable.

The findings in this study on PSTs' knowledge of points of inflexion were similar to the result of the study conducted by Tsamir and Ovodenko (2013), which showed that most of their students only located the stationary points of inflexion but failed to identify non-stationary points of inflexion. This could be explained by the earlier study conducted by Tsamir and Ovodenko (2004) that inflexion points were understood by most students as a point where the graph keeps increasing or decreasing (as demonstrated in T8's response) but dramatically changes the rate of change (or the slope). Carlson et al. (2002) asserted that students' acquisition of fragments of phrases about inflexion points could also be a cause of their incomplete knowledge of the concept. The PSTs' acquisition of incomplete knowledge of an inflexion point could also be explained by their concept of an inflexion point being a classification of stationary points as neither maximum nor minimum points without the notion of concavity as introduced in the secondary school mathematics curriculum (Singapore Ministry of Education (MOE), 2018).

The second derivative test for the nature of stationary points, which relates the concavity to the nature of stationary points, was first introduced in the secondary four calculus curriculum *procedurally* without the knowledge of concavity (MOE, 2018). This test was introduced merely as an additional procedural tool for the students, in addition to the first derivative test. Moreover, Orton (1983) asserted that the second derivative test is not a good tool to be introduced without conceptual understanding since the second derivative test does not always work, and in "later study specialists will meet many functions for which the procedure is inappropriate" (p. 244). Researchers have found it crucial to link the second derivative test to the underlying concept of concavity (e.g., Infante, 2016). Without this link being made explicit, the introduction of the second derivative test does not necessarily improve the PSTs' procedural fluency or flexibility in calculation tasks with the additional procedural tool. In the latest study by Toh (2022) on calculus instructional material developed by pre-university teachers, it was found that the link between the sign of the second derivative and the nature of stationary / inflexion points was not made explicit. Another study by Unver (2020) revealed that the pre-service teachers in his case study could not respond appropriately to students' questions related to second derivatives, which was likely due to their lack of the related content knowledge.

Conclusion

The Singapore mathematics curriculum for K to 12 can be described as spiral (Bruner, 1971). A concept that is visited at a lower level is re-visited iteratively at the higher levels with progressively increasing level of sophistication to develop a holistic understanding of the concept. The spiral approach to calculus from the secondary level to the pre-university level mathematics curriculum was illustrated in Toh (2021). In comparing the coverage of the concepts of stationary points and points of inflexion: the notion of maximum / minimum stationary point was covered completely at the secondary level while point of inflexion was first introduced spirally at the secondary level as a categorization of stationary points and later at the pre-university using the notion of concavity. Most of the PSTs did not identify the non-stationary inflexion points, similar to the study of Tsamir and Ovodenko (2013), but almost all the PSTs were able to successfully locate all the stationary points.

While researchers have recognised the advantages of a spiral curriculum, the result of this study suggests educators and curriculum designers might need to re-think how the spiral curriculum design could best be used for more advanced mathematics such as calculus. As early as 1980s, Orton (1983) cautioned that an imperfect knowledge introduced at an early stage might not be easily replaced by a more complete knowledge acquired at a later time, especially with reference to advanced concepts in calculus. Orton's caution perhaps poses a challenge for policymakers to re-examine the suitability of spiral approach to mathematics curriculum development for various strands of mathematics.

Recommendations

The school calculus within the mathematics curriculum should provide students with more opportunity to experience various types of functions with their multiple representations, in addition to procedural knowledge. In this vein of reasoning, graphs of functions which are not continuous or differentiable everywhere (in addition to the standard differentiable functions) should be introduced early in the school curriculum graphically. This will provide students with a flexible and robust view of functions. We agree with Koirala's (1997) opinion that an introductory calculus course should be informal, intuitive and conceptually based mainly on graphs and functions, which was further supported Heid (1988), Orton (1983) and Parameswaran (2007). The existing school calculus course focuses on procedural knowledge. Perhaps an ideal introductory course at the secondary or pre-university level should have a better alignment of the iconic thinking and procedural knowledge for related calculus concepts. Note that the importance of iconic thinking cannot be underestimated as it will have impact on students' reasoning on calculus concepts (e.g., Nurwahyu et al., 2020).

The result of this study could be helpful for university lecturers to re-examine the design of undergraduate level calculus courses. Any formal thinking in the undergraduate calculus courses should be able to build on the students' iconic thinking and procedural knowledge related to the calculus concepts. Much of this design could be to focus on strengthening the connections between the school calculus and undergraduate calculus. This is particularly important to facilitate pre-service teachers to develop sound content knowledge at the undergraduate level in order to appreciate school mathematics content.

Although this study has its limitation, we hope that this study could spur further interest among researchers on PSTs' content knowledge on calculus and how their calculus content knowledge is impacted by their school calculus content, and their eventual calculus content knowledge after they have completed the teacher education course.

Limitations

Due to the constraint of the design, neither an interview with selected participants nor a focused group discussion was feasible. To compensate for this, the participants were encouraged to provide comment for each of the items in the calculus survey. However, it was not mandatory for them to provide comment, hence a significant number of them did not provide any comment for some items. We were not able to obtain full information about their knowledge.

The participants in this study were not characteristic of all the PSTs in the entire cohort of the teacher education programme over the years. The group of PSTs participating in this study was selected under very stringent condition from the entire cohort of pre-university students with very good academic results, in particular, with very high scores in the GCE O-Level and A-Level examinations (the high-stake national examinations at the end of secondary and pre-university education respectively), particularly in mathematics. Thus, their performance might not be characteristic of all PSTs in the country, but could be considered as the performance of the upper bound of the student cohort. Moreover, as all the participants had knowledge of calculus up to the pre-university level, this study could serve as an indicator of the PSTs' knowledge of school calculus. Despite the limitation of this study, we hope that this study spurs further interest among more researchers on students' and teachers' calculus content knowledge and results in further research into this area.

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Authorship Contribution Statement

Toh T.L.: Contributed to the conceptualization, design, data analysis, and writing in the first draft. Toh P.C., Teo and Zhu: Contributed to the conceptualization, design, data analysis, editing the first draft and reviewing of the manuscript.

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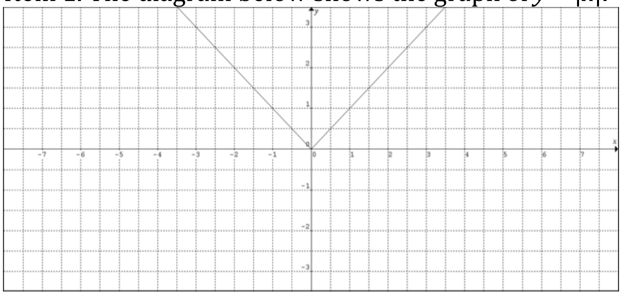
Appendix

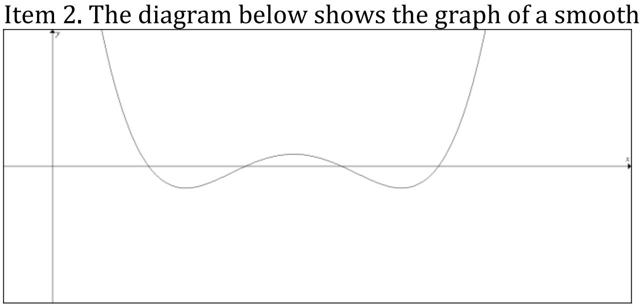
Item 1. The diagram below shows the graph of $y = |x|$.



Which of the following is true about the point (0, 0) in the above graph?

(A) It is a minimum point.
(B) It is a stationary point.
(C) It is not a turning point.
(D) At the point (0, 0) on the graph, $\frac{dy}{dx} = 0$.
(E) None of the above statements is TRUE.

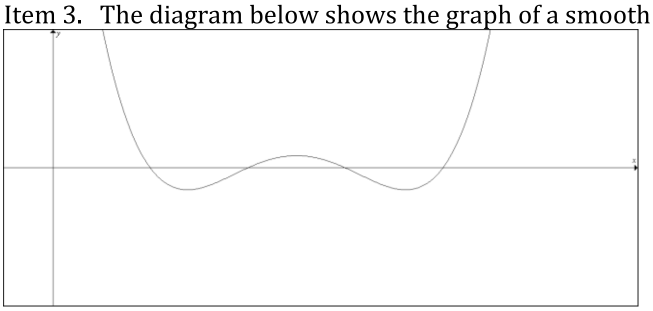
Item 2. The diagram below shows the graph of a smooth function.



How many stationary points are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Item 3. The diagram below shows the graph of a smooth function.



How many points of inflexion are there?

(A) 0 (B) 1 (C) 2 (D) 3 (E) More than 3

Item 4. Consider a function f. If for x < a, we have f(x) < f(a) and for all x > a, we have f(x) < f(a), then what can you say about the point (a, f(a))?

(A) It is definitely a maximum point.
(B) It could be a maximum point, but we cannot conclude unless we use first derivative or second derivative test.
(C) It could be a maximum point, but we cannot conclude since the function is not given to us hence we cannot differentiate.
(D) It is an increasing function.
(E) It is a maximum point but it may not give the absolute maximum value of f.

Item 5. The gradient function of a graph is given by $\frac{dy}{dx} = (x-1)^7\sqrt{1+x^{10}}$. What can you tell about the point $x = 1$ on the graph?

(A) At $x = 1$, it is a maximum point.
(B) At $x = 1$, it is a minimum point.
(C) At $x = 1$, it is a stationary point of inflexion.
(D) At $x = 1$, it is a stationary point but there is insufficient information to justify the nature of this stationary point.
(E) At $x = 1$, it may not be a stationary point.