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Exploration of Prospective Mathematics Teachers' Mathematical Connections When Solving the Integral Calculus Problems Based on Prior Knowledge

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Abstract: Mathematical connection ability is very important to be mastered by prospective mathematics teacher students as competency to teach in secondary schools. However, the facts show that there are still many students who have weak mathematical connection abilities. This qualitative descriptive study aimed to explore how the process, and product of the mathematical connection made by prospective mathematics teacher students when solving the integral calculus problems based on their prior knowledge. The research subjects were 58 students who were prospective high school mathematics teachers at the University of Jember, Indonesia. Data were collected using documentation, questionnaire, test, and interview methods. After the test results of all subjects were analyzed, six students were interviewed. To find the match between the results of the written test and the results of the interview, a triangulation method was carried out. Data analysis used descriptive qualitative analysis with steps of data categorization, data presentation, interpretation, and making conclusions. The results show that the research subjects have connected and used mathematical ideas in the form of procedures, facts, concepts/principles, and representations in solving integral calculus problems. Students with high prior knowledge abilities can make better mathematical connections than students with moderate and low prior abilities. From these results, it is recommended that lecturers need to improve students' prior knowledge and train the students more intensely to solve integral calculus problems so all students can develop their mathematical connection abilities into very strong categories.

Keywords: *Integral calculus, mathematical connection, prior knowledge, process, and product.*

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Introduction

Mathematical connection ability is very important for prospective mathematics teachers because they will later become teachers and will teach the basics of Integral Calculus (IC) in high school. But based on teaching experience so far, not all students have strong mathematical connection abilities and there are still many students who have weak mathematical connection abilities. Even though the current condition shows that the mathematical connection ability of students in Indonesia is still very low. For example, research results show that elementary school students' mathematical connection abilities are still low (Kenedi et al., 2019). Siregar and Surya (2017) emphasize that only reached an average of 51.11%. Thus, after becoming mathematics teachers, it is hoped that they will be able to improve students' Mathematical Connection (MC) abilities.

Several factors affect students' mathematical connection skills when solving IC problems, including students' prior knowledge. Prior knowledge is the knowledge that a person has from his learning experience before he/she learns new knowledge. This research shows that students' prior knowledge greatly influences individual success in learning new knowledge, especially those related to previous knowledge. For example, to learn concept B, students need to master concept A which underlies concept B (Arifin, 2019; Dong et al., 2020; Hailikari et al., 2008; Rittle-Johnson et al., 2009). When studying Differential Calculus (DC) courses, students learn the concept of Derivatives which is the basis for learning the concept of Integral. Therefore, the success of students in studying IC courses can be determined by the ability of students to master the concept of Derivatives, in this case, the value of DC courses.

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This research that examines urgent mathematical connections is carried out in the IC course, because there is not much research on IC (García-García & Dolores-Flores, 2018, 2020, 2021; Meneses et al., 2021; Nieto et al., 2021, 2022). For example, Ferrer (2016), and Seah (2005) examined students' difficulties in working on Integral questions. Eli et al. (2011) examined the mathematical connections of prospective secondary school teachers using task cards; Moon et al. (2013) examined students' difficulties in making mathematical connections; Kouropatov and Dreyfus (2014); Jones (2015) examined the area; Radmehr and Drake (2017) examined the concept of Basic Calculus; García-García and Dolores-Flores (2018, 2020, 2021), Meneses et al. (2021), and Nieto et al. (2021, 2022) researched the concepts of functions, graphs, and derivatives. Moreover, research that examines prior knowledge related to mathematical connections in IC problem solving is still very rare. In addition, Rasmussen and Wawro (2017) said that research on teaching and learning calculus is very interesting and important to do. Therefore, it is necessary to explore how the process and product of the mathematical connection made by prospective high school mathematics teachers on the problem of IC problems based on their prior knowledge.

This study aims to explore how the process and product of mathematical connections made by prospective mathematics teachers when solving problems of applying IC based on their prior knowledge. The questions asked are as follows:

1. What are the quantity, quality, and strength of the product of mathematical connections made by prospective mathematics teachers when solving the IC problems based on their prior knowledge?
2. What are the processes of prospective mathematics teachers in producing mathematical connections when solving the IC problems based on their prior knowledge?

The expected benefits of this research can provide input to lecturers, prospective mathematics teacher students, and other stakeholders regarding the process and product of mathematical connections made by prospective high school mathematics teachers on the IC problems based on their prior knowledge. In addition, to provide information on the need for further research on mathematical connections.

Literature Review

Mathematical Connection

Mathematical connection is the relationship between ideas in mathematics, the relationship between mathematical ideas with other fields; and the relationship between mathematical ideas and real-life problems (García-García & Dolores-Flores, 2018, 2020, 2021; Hatisaru, 2022; National Council of Teachers of Mathematics [NCTM], 2000, 2014; Toh & Choy, 2021). A mathematical connection can be thought of as part of a mental network structured like a spider's web. The points or nodes can be thought of as pieces of information, and the threads between them as the connections, so that all nodes on the network are always connected. The strength of the connection is influenced by the number of networks, and the accuracy of the network connections (Eli et al., 2013; Hiebert & Carpenter, 1992; Pambudi et al., 2020).

It is very important to teach mathematical connections to students, from basic education to higher education, so that they realize that mathematics is a science that is integrated into a single unit, and not a collection of separate materials (NCTM, 2000, 2014; Sawyer, 2008). In addition, students can learn mathematics better if the material studied is related to the ideas that have been studied previously, and provides a deep understanding and embedded in students' long-term memory (García-García & Dolores-Flores, 2018, 2020, 2021; Hatisaru, 2022; NCTM, 2000, 2014). In addition, mathematical connection has a role as an effective tool in mathematical problem-solving activities (NCTM, 2000, 2014; Pambudi, 2020). Students with good mathematical connection skills will succeed in solving mathematical problems, and conversely, students with weak mathematical connection abilities will fail to solve problems (Altay et al., 2017; Michigan State University, 2012; Pambudi, 2020). Therefore, it is very important for teachers to always guide students to make mathematical connections, especially when students solve mathematical problems (Anthony & Walshaw, 2009; Arthur et al., 2018).

Teachers need to make students aware that although mathematics consists of various objects or ideas, such as procedures, facts, concepts/principles, representations, and procedures, all these ideas need to be linked to be able to learn more difficult material (Lappan et al., 2002; Thompson, 2008). Baki et al. (2009) stated that to solve mathematical problems, students should understand the problem and make mathematical connections between various ideas or objects in mathematics, which include facts, procedures, concepts/principles, and representations either in the form of verbal (sentences), graphs, tables, pictures, symbols, equations, and mathematical operations.

Processes and Products of the Mathematical Connections

The activity of students making mathematical connections when solving math problems is very interesting to study because both are the goals of learning mathematics (Liljedahl et al., 2016; Minister of Education and Culture [MoEC], 2016). In this activity, it can be studied how the process of students making mathematical connections, and how the resulting mathematical connection products. The process studied is the mental process that students do when connecting and using connections of mathematical ideas in solving mathematical problems (Baiduri et al., 2020; Eli et al., 2011, 2013; Hatisaru, 2022; Mhlolo et al., 2012; Pambudi, 2020).

The product of mathematical connections produced by students can be seen from the type and strength. This type of mathematical connection can consist of links between mathematical ideas in the form of mathematical ideas, procedures, facts, concepts/principles, and representations (Pambudi et al., 2020). The connection quantity is the number of mathematical ideas that students relate to, and the quality of the connection is the accuracy of the association of mathematical ideas that are made and used to solve problems. The strength of mathematical connections is seen in the quantity and quality of mathematical ideas associated with solving problems (Eli et al., 2013; Pambudi et al., 2020).

Integral Calculus

Calculus is a branch of mathematics that is divided into Differential Calculus (DC) and Integral Calculus (IC). These two courses are made mandatory courses for prospective high school mathematics teacher students, which are taken in semesters 1 and 2. DC courses are before taking IC courses. In IC courses, students learn the definition of integral as anti-derivative, various methods for solving integral problems, and the application of IC. The material studied in the application of IC includes calculating the area between two curves (Varberg et al., 2013).

Prior Knowledge and its Relationship to Learning Achievement in Mathematics

Prior knowledge is needed by someone who is learning new knowledge related to previous knowledge. The ability of students to master prior knowledge has a very close relationship with students' mathematical connections (Siagian et al., 2021; Sidney & Alibali, 2015), and with student learning outcomes (Hailikari et al., 2008; Rittle-Johnson et al., 2009). Students who have good prior knowledge tend to have greater potential for success in learning new knowledge than students who have poor prior knowledge (Arifin, 2019; Dong et al., 2020; Ningsih & Retnowati, 2020; Oyinloye & Popoola, 2013; Simonsmeier et al., 2021). Therefore, mathematics material at the school and university level is structured in a structured manner, starting from prerequisite material to more advanced material, and so on. Thus, it is expected that students can learn the material smoothly and well.

About to with concerning the IC course, the previous knowledge that is a prerequisite for taking the IC course is the DC course obtained by the student in semester 1. The student's ability to master DC knowledge is indicated by the scores of the DC course obtained by the student. The scores are in the intervals A(80-100), AB(75-79.99), B(70-74.99), BC(65-69.99), C(60-64.99), CD(50-59.99), D(40-49.99), and E(0-39.99) (FKIP University of Jember, 2014).

When solving the problem of calculating the area between two curves, there are several steps needed to be carried out by students, namely (1) sketching a graph of the area to be calculated; (2) finding the point of intersection between two curves; (3) cut/slice the shaded area, and approximate the area; and (4) calculate the area using the definite integral theorem (Varberg et al., 2013). From these four steps, it can be studied whether students perform all of these steps in the process of making connections or only do some of them. Then, how the processes and connection products that make students successful in solving these problems. Based on the description above, this research uses a research framework as presented in Figure 1.

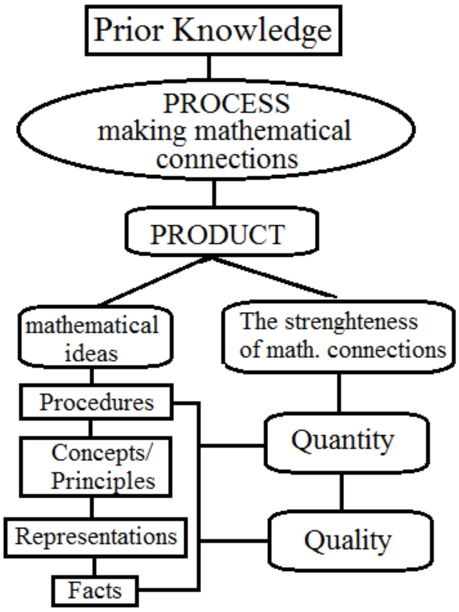

Figure 1. Research Framework

Methodology

Research Design

The aim of this research is to explore how the process and product of mathematical connections made by prospective mathematics teachers when solving problems of applying IC based on their prior knowledge. Therefore, the descriptive research was conducted using a qualitative approach.

Sample and Data Collection

The research subjects in this study were third-semester students, and prospective high school mathematics teachers who had taken the Integral Calculus course in the second semester of 2019/2020 at the University of Jember, Indonesia. The number of students whose data was taken was 58 people, with an average age of 20 years, who came from two classes, namely class B (n=35) and class E (n=23). Data were collected using documentation, questionnaire, test, and interview methods. Data on the scores of DC courses as prior knowledge to take IC courses are obtained from student Study Results documents. The written test consists of two tasks regarding the area between two curves, namely "(1) Calculate the area between the line $y=6x$, and the curve $y=6x^2$; and (2) Calculate the area between the line $xy-1=0$, and the curve $x+y^2-3=0$" (Varberg et al., 2013). Both tasks were recognized as valid (Va = 2.75 in score validation of between 0 to 3.00 interval), and reliable (r = 0.80). The tasks are given to all students, then the results were corrected and analyzed. Interviews were conducted with 6 students, namely representatives of students with lowest, moderate, and highest prior knowledge, two each. Interviews were conducted to verify and find out how the subject process when doing the written test. To obtain valid data, the triangulation method is carried out, namely looking for conformity between the results of the written test and the results of the interview.

Analyzing of Data

Data analysis used descriptive qualitative analysis with steps of data categorization, data presentation, interpretation, and making conclusions (Miles et al., 2014; Moleong, 2013; Pambudi, 2020). In the data categorization step, data or information classification is carried out according to process indicators and mathematical connection products. The presentation of the data is done with tables, percentages, and figures according to the process and product of the mathematical connections made by the subjects. The next interpretation is carried out from the results of triangulation of written test data and interview results, followed by making conclusions.

Findings / Results

Prior Knowledge and Integral Calculus Problems Test Scores

Prior Knowledge data obtained from the document scores of the DC course and IC problem test scores from research subjects. The interval scores and the percentages of prior knowledge from 58 subjects can be seen in Table 1.

Table 1. Prior Knowledge and Integral Calculus Problems Test Scores

Interval of Scores	Percentage (%) N=58		
	Prior Knowledge	**Task Number 1**	**Task Number 2**
A	10.34	51.72	18.97
AB	27.59	0.00	0.00
B	32.76	48.28	65.52
BC	3.45	0.00	0.00
C	22.41	0.00	6.89
CD	0.00	0.00	5.17
D	3.45	0.00	3.45
DE	0.00	0.00	0.00
E	0.00	0.00	0.00
Total	**100.00**	**100.00**	**100.00**

From Table 1, it can be seen that the majority of subjects had good prior knowledge. There are 32.76% scored B, 27.59% scored AB, 22.41% scored C, 10.34% scored A, and only 3.45. % of students who have poor prior knowledge, namely getting score D.

The Quantity and Quality of Mathematical Ideas in Mathematical Connections Made by Research Subjects

The mathematical connections made by the research subjects when working on two IC problems in the area between the two curves were analyzed based on the adequacy and accuracy of the mathematical ideas associated and used in solving problems, as well as how strong the connection of these mathematical ideas was. This can be seen in Table 2.

Table 2. The Quantity and Quality of Mathematical Ideas in Mathematical Connections (MC) Made by Research Subjects

Types of MC	The Quantity and Quality of Mathematical Ideas in MC	
	Task Number 1	**Task Number 2**
Type I	MC = Pro + F + C/P + Rep Pro = Step1 + Step2 + Step3 + Step4 F = F1 + F2 C/P= C/P1+C/P2+C/P3+C/P4+C/P5 Rep= Rep1+Rep2+Rep3+Rep4+Rep5	MC = Pro + F + C/P + Rep Pro = Step1 + Step2 + Step3 + Step4 F = F1 + F2 C/P= C/P1+C/P2+C/P3+C/P4+C/P5 Rep= Rep1+Rep2+Rep3+Rep4+Rep5
	Very Sufficient and Very Precise	**Very Sufficient and Very Precise**
Type II	MC = Pro + F + C/P + Rep Pro = Step2 + Step4 F = F1 + F2 C/P= C/P2+C/P3+C/P4+C/P5 Rep= Rep1+Rep3+Rep4+Rep5	MC = Pro + F + C/P + Rep Pro = Step2 + Step 3 + Step4 F = F1 + F2 C/P= C/P2+C/P3+C/P4+C/P5 Rep= Rep1+Rep3+Rep4+Rep5
	Sufficient and Precise	**Sufficient and Precise**
Type III	None	MC = Pro + F + C/P + Rep Pro = Step2+Step4 F = F1 + F2 C/P= C/P2+C/P3 Rep = Rep1+Rep3
		MC = Pro + F + C/P + Rep Pro = Step1+Step2+Step4 F = F1 + F2 C/P= C/P1+C/P2+C/P3 Rep= Rep1+Rep2+Rep3
		Insufficient and not Precise

From Table 2, it can be seen that the research subjects as many as 58 people have produced mathematical connections when solving two tasks which are divided into 3 types, namely (1) mathematical connections with a very sufficient number of mathematical ideas and the association of mathematical ideas is very precise, (2) connections mathematical connection with a sufficient number of mathematical ideas and the association of mathematical ideas is appropriate, and (3) mathematical connection with less number of mathematical ideas, and the association of mathematical ideas is not appropriate.

Mathematical ideas that are associated and used by the subject in solving problems include procedures, facts, concepts/principles, and representations. The complete procedure (Pro) performed is Step 1 (Sketch Graph), Step 2 (Find Intersection Points between two curves), Step 3 (Slice and Approximate), and Step 4 (Calculate the Area). All subjects relate and use facts (F) obtained from the problem, namely F1 (line equation), and F2 (curve equation). The concepts/principles (C/P) that are associated and used by the subject consist of C/P1 (Sketch Graph, line, and curve), C/P2 (Cartesian Coordinate), C/P3 (Quadratics Equation), C/P4 (Finite Integral), and C/P5 (Operations in Mathematics). The representations (Rep) that are associated and used by the subject include Rep 1 (Graphics (line & curve), Rep 2 (Symbols), Rep 3 (Equation), Rep 4 (Operation in Mathematics), who use ideas, but some only use part of them to solve problems. For example, some subjects only perform the procedures Step 2 and Step 4, or Step 2, Step 3, and Step 4. Likewise, the same thing happened to concepts/principles, and representations.

Based on the information on the quantity and quality of the mathematical connection product, the strength of the mathematical connection product made by the research subjects when solving the problems was obtained. Table 3 presented the percentage of strengthens of mathematical connections (MC) from all subjects.

Table 3. Strength of the mathematical connection products made by subjects

Task	Class	Percentage of Strengthens of MC (%) (n=58)			Total (%)
		VSC	SC	WC	
Number 1	Class B (n=35)	46.55	13.79	0.00	100
	Class E (n=23)	5.17	34.49	0.00	
Total (%)		51.72	48.28	0.00	
Number 2	Class B (n=35)	17.24	36.21	6.90	100
	Class E (n=23)	1.72	29.31	8.62	
Total (%)		18.96	65.52	15.52	

From Table 3, it can be seen that the research subjects were able to make or produce mathematical connections in three categories, namely very strong connections (VSC), strong connections (SC), and weak connections (WC). In task number 1, 51.72% of research subjects were able to make connections to mathematical ideas in the very strong category, followed by 48.28% in the strong connection category. In task number 2, 65.52% of research subjects were able to make connections to mathematical ideas in the strong connection category, followed by 18.96% in the very strong connection category, and 15.52% in the weak connection category.

The Process of Producing Mathematical Connections Made by Highest Prior Knowledge Students

Of all research subjects, two subjects with the highest prior knowledge can produce mathematical connections with the Very Strong category for Task number 1, and Task number 2. Examples of products with the Very Strong category made by subject S1 and subject S2 can be seen in Figure 2.

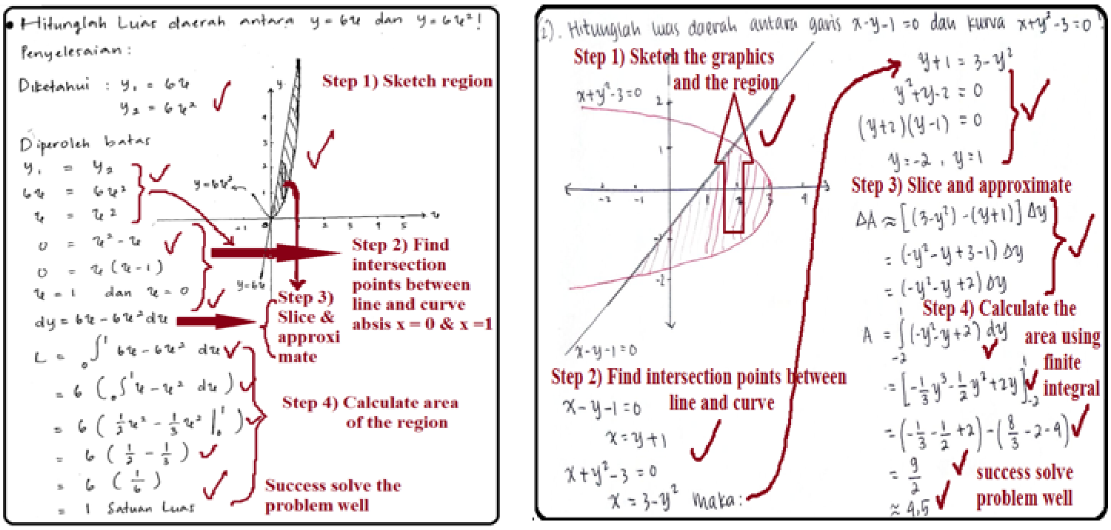

Figure 2. Examples of Very Strong Mathematical Connection Products Made by Subjects S1 and S2 (left for task number 1, and right for task number 2)

To find out the process of S1 and S2 subjects when producing mathematical connections as shown in Figure 2, interviews were conducted. A snippet of the interview can be seen below. (R = Research, S1 = Subject S1, S2 = Subject S2).

R : What do you do after reading the question?

S1 : After receiving the question, I wrote the question, then I thought for a while, I remembered the Integral Calculus course, which is calculating the area between two curves, then I answered the question.

S2 : I read the problem, I think it's about Integral Calculus. Then I remembered the steps for calculating the area between two curves.

R : Can you explain the steps to answer this question?

S1 : I write problems and write equations of lines and curves. Then I draw lines and curves, ... then I shade the area to be calculated. Then I look for the point of intersection between the line and the curve. Then I took the slice vertically and made an approximation of dy. After that, I calculated the area with the finite integral formula.

S2 : I sketched lines and curves according to the equation of the problem. Then I shaded the area to be calculated. Next, I look for the point of intersection between the line and the curve. This is to obtain a lower bound and an upper bound for calculating the integral. Then I make an approximation of the area, which is A. Then I calculated the area with the finite integral formula.

The two subjects understand all the mathematical ideas associated with answering the questions. For example, the researcher asked the subject S2 why he chose the limit of integration of y instead of x. He replied because I see it is easier to calculate the area of the region horizontally. He could calculate the area vertically, but he had to divide the shaded area into two halves, left, and right, so that would require two integrals, and that would take longer. When asked what the unit of area was, the subject answered the unit of area, but he forgot to write the unit on the answer sheet.

From Figure 2, it can be said that the subjects of S1 and S2 relate and use various mathematical ideas, namely in the form of procedures, facts, concepts/principles, and representations in solving problems. The facts used are line equations and quadratic curve equations obtained from the problem. The procedure carried out is to carry out four stages, starting from step 1 (Sketch Graph), step 2 (Find Intersection Points) between line and curve, step 3 (Slice and Approximate), to step 4 (Calculate the Area). The concepts/principles that are associated and used are sketch graph (line & curve), cartesian coordinates, quadratics equation, finite integral, and some operations in mathematics. The representations used are graphics (line & curve), symbols, equations, and some operations in mathematics. All the mathematical ideas used are sufficient in number, and all of them are linked correctly so that the subjects S1 and S2 managed to solve the problem of the area between two curves correctly. After both subjects got the answer to the problem, then they ended the process. From this, it can be said that the subject of S1 and the subject of S2 already have a mathematical connection with the Very Strong category.

The Process of Making Mathematical Connection Products Made by Moderate Prior Knowledge Students

Of all research subjects, the subjects with moderate prior knowledge can produce mathematical connections with the Strong category. The examples of strong mathematical connection products made by two subjects can be seen in Figure 3.

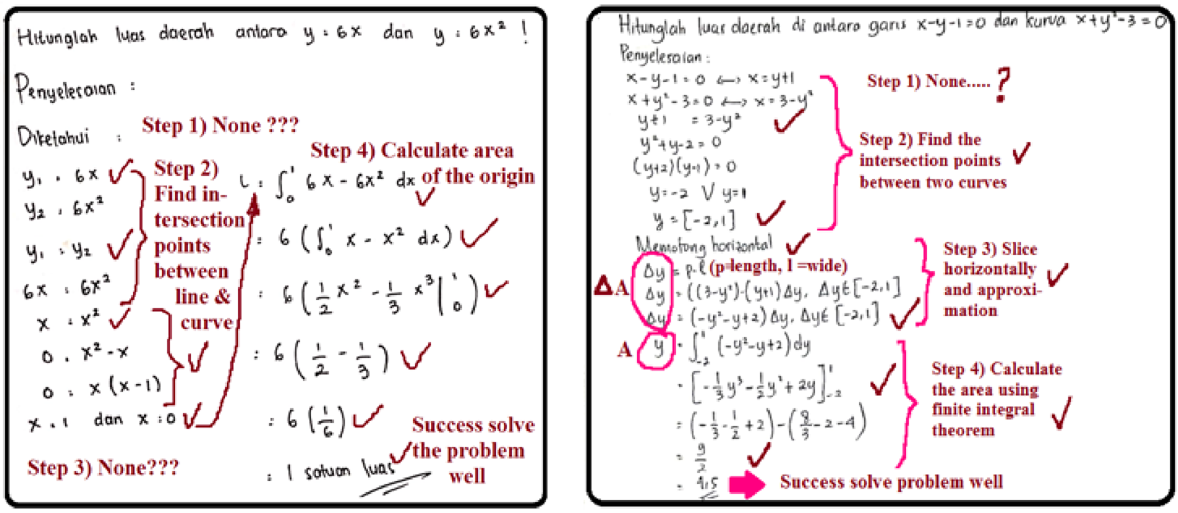

Figure 3. Examples of Strong Mathematical Connection Products Made by Subjects S3 and S4 (left for task number 1, and right for task number 2)

To find out the process of S3 and S4 subjects when producing mathematical connections, interviews were conducted. A snippet of the interview can be seen below. (R = Research, S3 = Subject S3, S4 = Subject S4).

R : Tell me what did you do after reading the question?

S3 : I read the question, then I wrote it and wrote what I knew, namely the equations of lines and curves. Then I answered the question.

S4 : I read the question, and I immediately answered the question.

R : Please explain what steps you took to answer this question?

S3 : First, I look for the point of intersection between the line and the curve. After I get the limit of integration, then I calculate the area between the line and the curve with a finite integral. After I calculated I got the area of 1 unit area.

S4 : I'm looking for the intersection of the line and the curve to get the lower and upper bounds of the integration. Then I make an approximation of the area. Then I calculated the area with the finite integral formula. After I calculate I get 4.5 areas.

Neither subject S3 nor subject S4 did not sketch the graph but instead looked for the point of intersection between the line and the curve. When asked, they answered that it was better to draw a sketch first, but that it would take a long time. When asked how are you sure your answer is correct if you don't know the area to be calculated. Their answer is because they have got the limit of integration, where question number 1 uses the limit of integration x, and number 2 uses the limit of integration y.

Based on the results of written tests and interviews, the subject of S3 and S4 have used various mathematical ideas, namely in the form of procedures, facts, concepts, principles, and representations. The difference with S1 and S2 subjects is that S3 and S4 subjects only perform part of the procedure. Subject S3 only performed the procedure step 2 and step 4. Subject S3 did not draw a graphic sketch, but he immediately carried out the second procedure, which was to find the point of intersection between the line and the curve. Next, the subject of S3 performs the fourth procedure, which is to calculate the area with the definite integral theorem. The S4 subject performed the second, third, and fourth procedures, and this was better than the S3 subject. The weakness of the subject of S4 is that he uses the wrong symbol in the procedure for making approximations, y should be A, and he does not write the unit of area in the final answer. Although S3 and S4 subjects did not carry out all standard procedures, all the essential mathematical ideas were sufficient in number, and all of them were properly linked and used effectively, namely finding the intersection point between a line and a curve, then calculating the area of the area using the finite integral concept, and performing operations. From this, it can be said that the S3 subject has a mathematical connection with the Strong category.

The Process of Making Mathematical Connection Products Made by Lowest Prior Knowledge Students

Of all research subjects, the subjects' lowest prior knowledge can produce mathematical connections with weak categories. The examples of weak mathematical connection products made by two subjects as can be seen in Figure 4.

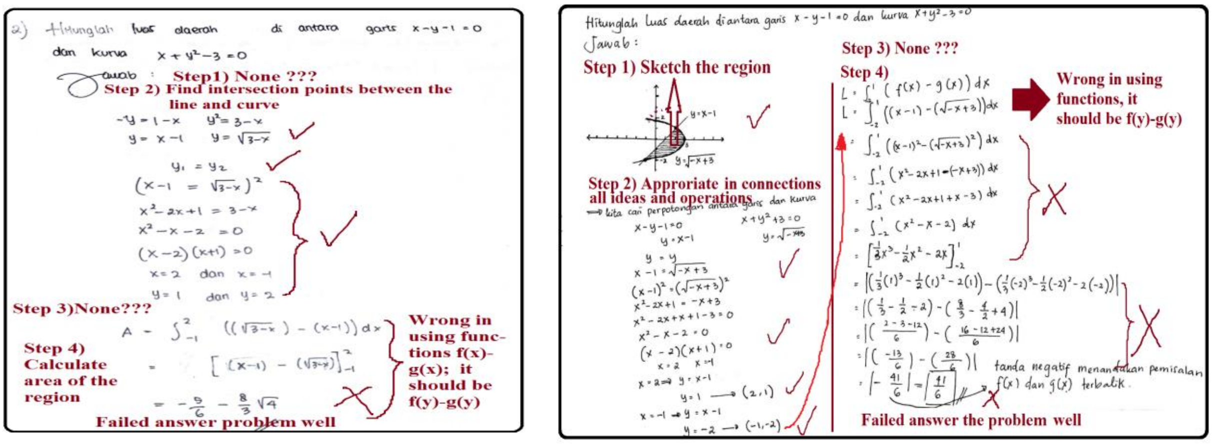

Figure 4. Examples of Weak Mathematical Connection Products Made by Subjects S5 and S6 (left for task number 1, and right for task number 2)

The following is a snippet of interviews with subjects S5 and S6. (R = Research, S5 = Subject S5, S6 = Subject S6).

R : Tell me what did you do after reading the question?
S5 : I write the question, then I answer the question.
S6 : I read the question, then I wrote the question. After that, I answered the question.
R : Please explain how the steps you took to answer this question?
S5 : I'm looking for the intersection point between a line and a curve. After I get the limit of integration, then I calculate the area with finite integral. After I calculated I got the area like this (while pointing to the answer on the paper).
S6 : I drew lines and curves, then shaded the area to be calculated. Then I look for the point of intersection between the line and the curve. After that, I calculated the area with the finite integral formula. The result is like this (pointing to the answer on the paper).

From the results of the written test and interview, the subject of S5 did not sketch a graph but immediately looked for the point of intersection between the line and the curve. Next subject S5 looks for the point of intersection between the line and the curve. However, when looking for an area, subject S5 uses the function *f(x)-g(x)*, and this is a fatal error, so the final answer is wrong. The S6 subject is better than the S5 subject because it sketches the graph, then looks for the intersection point between the two curves. Unfortunately, subject S6 made the same mistake as subject S5, by using the *f(x)-g(x)* function, and as a result of this fatal error, the final answer was also wrong.

Based on the types of mathematical ideas connected to subject S5, it turns out that subject S5 only performs two procedures, namely the second and fourth steps. In the second step, all mathematical ideas are linked correctly, so that the coordinates of the point of intersection between the line and the curve are obtained. However, in the fourth step, subject S5 made an error, namely using the *f(x)-g(x)* function, and this connection was incorrect (wrong), so she failed to solve the problem. Meanwhile, subject S6 was better than subject S5, when she performed the first, second, and fourth procedures. In the first step, she was able to draw graphic sketches well and continued to find the intersection point between lines and curves well too. Unfortunately, subject S6 relates and uses the function *f(x)-g(x)* when calculating the area, when it should be *f(y)-g(y)*. From this, it is clear that the number of essential mathematical ideas associated with the two subjects is less (not enough), and the association of these mathematical ideas is incorrect, especially in step 4. The impact of this is that they fail to answer the problem. From this, it can be said that subject S5 and subject S6 still have a mathematical connection with the Weak category.

Discussion

Research subjects have varying prior knowledge, where there are very good DC scores, which can be A, good (AB, and B), moderate (C), and poor (D). With the difference in prior knowledge, it turns out that it also makes a difference in the ability to solve IC problems. Where students who have good prior knowledge succeed in solving IC problems, while students who have poor prior knowledge fail to solve the problems. These results are in accordance with the results of research conducted by Arifin (2019); Dong et al. (2020); Hailikari et al. (2008); Rittle-Johnson et al. (2009); Simonsmeier et al. (2021). In connection with the task in this research, the success of students in solving IC problems is influenced by the mastery of concepts/principles in the DC course. The concepts/principles in prior knowledge needed are the ability to make graphs, determine the point of intersection between two curves, determine the area to be searched for, and determine the function to be integrated. Therefore, at the beginning of the lecture, the lecturer should make an apperception to remind the DC concepts needed to master IC. Thus, students' prior knowledge can be raised again so that students are ready to take part in the learning process in IC courses.

Of the two tasks given, it turned out that the second question was more challenging than the first. Where, in the first question, the subject was able to produce mathematical connections with a very strong category (51.72%), a strong category (48.28%), and none with a weak connection; while in the second question, the research subjects were able to make mathematical connections with the category of strong connection (65.52%), very strong (18.96%), and weak (15.52%). In the first problem, the lines and curves have an explicit function, namely $y=6x$, and $y=6x^2$, while the second question has an implicit function, namely $x-y-1=0$, and $x+y^2-3=0$. The subject is easier to draw a graphic sketch of the first question than the second question. Another reason is that the area to be calculated in the first question is in the first quadrant, while in the second question, the area to be calculated is more difficult because it is in the first, third, and fourth quadrants. Another challenge is that the second question requires more accuracy than the first question. Where the area in the second problem can be calculated by cutting (slice) horizontally. Here some subjects who are not careful have an error in choosing the limit of integration or the function to be integrated. In the first problem, the subject is easier to calculate the area by cutting vertically, and using the function $y=f(x)-g(x)$ to be integrated. In the second question, students have to cut horizontally and use the function $x=f(y)-g(y)$ as the integrator, and not the function $y=f(x)-g(x)$. The subject's errors in representation, in particular drawing graphs, using symbols, determining functions as integrals, formulating integrals, performing operations in integrals are also recognized by Ferrer (2016), Moon et al. (2013), and Radmehr and Drake (2017), and Seah (2005).

Generally, prospective high school mathematics teachers who are the subjects of this study connect and use mathematical ideas in the form of procedures, facts, concepts/principles, and representations in solving the problem of the area between two curves. In particular, the difference in prior knowledge also makes a difference in the quantity and quality of students' mathematical connections in solving IC problems. Where students who have good prior knowledge tend to be able to produce strong mathematical connections, and vice versa. Students who have poor prior knowledge tend to produce weak mathematical connections. This is reasonable because students who have good prior knowledge can connect various mathematical ideas that they have mastered well to solve problems, while students who have poor prior knowledge will lack ideas to connect in solving problems. These results are suitable with research conducted by Siagian et al. (2021); and Sidney and Alibali (2015).

The quantity and quality of the mathematical connection of these mathematical ideas consist of 3 types, namely very sufficient and very precise, sufficient and precise, and little/less and not precise. The strength of the mathematical connections produced by the subject consisted of three categories, namely very strong, strong, and weak categories. This shows that the strength of mathematical connections is influenced by the quantity and quality of connections of mathematical ideas used in solving problems. These results are by following with the theory and results of research (Eli et al., 2013; Hiebert & Carpenter, 1992; Pambudi et al., 2020).

There are three kinds of processes and produce three types of mathematical connection products when the subject solves the area problem between two curves. First, subjects who carry out the process of making mathematical connections by performing complete procedures, starting from step 1, step 2, step 3, and step 4. Subjects who carry out complete procedures can produce mathematical connections with very strong categories, and successfully solve problems correctly. Second, subjects who perform the step1 procedure, step 2, and step 4 or only step 2, and step 4; but use all the essential mathematical ideas which are sufficient in number and the association of all the essential ideas is all correct so that they can solve problems correctly. These essential mathematical ideas are those included in Step 2 and Step 4 procedures. This second group belongs to the category of Strong mathematical connections. Third, the subjects who performed incomplete procedures, used fewer numbers of mathematical ideas and the association of essential ideas was not all correct, and as a result, they failed to solve the problem. This third group is included in the category of weak mathematical connections. So, the success of students in solving mathematical problems can be influenced by the strength of mathematical connections. This is by following the theory and research results of Altay et al. (2017); Eli et al. (2013); Michigan State University (2012); NCTM (2000, 2014), and Pambudi et al. (2020).

Conclusion

Generally, prospective high school mathematics teachers who are the subjects of this study connect and use mathematical ideas in the form of procedures, facts, concepts/principles, and representations in solving the Integral Calculus problems. Students who have good prior knowledge tend to be able to produce strong mathematical connections, and students who have poor prior knowledge tend to produce weak mathematical connections. There are three types of processes and produce three types of mathematical connection products when the subject solves the problems. First, the subjects who carried out the process of connecting all procedures in a very sufficient number and the connections were very precise, so they were able to produce mathematical connections with very strong categories. Second, the subjects who carried out the process of connecting some procedures, but the essential mathematical ideas that were connected were sufficient and the connections were correct so that they were able to produce mathematical connections with strong categories. Third, the subjects who carried out the process of connecting some procedures, and using fewer numbers of mathematical ideas and the connection of essential ideas were wrong, resulting in a product of weak mathematical connections. The quantity and quality of the mathematical connection of these mathematical ideas consist of 3 types, namely very sufficient and very precise, sufficient and precise, and little/less and not precise. The strength of the mathematical connections produced by the subject consisted of three categories, namely very strong, strong, and weak categories.

Recommendations

From these results, it is recommended that lecturers need to do apperception to remind the DC concepts needed to master IC. Thus, students' prior knowledge becomes good so that students are ready to follow the learning process in IC courses. Second, lecturers need to train students more intensely to solve Integral Calculus problems so all students can develop their mathematical connection abilities into a very strong category. This research needs to be continued by taking more subjects, and the material provided is more varied and more difficult. In addition, it is necessary to develop the management of the Integral Calculus course, and it is necessary to develop a rubric for measuring the level of mathematical connection strength which is more varied, starting from the very strong, strong, medium, weak, and very weak categories as a reference for researchers in assessing the level of mathematical connection ability.

Limitations

The weakness of this study is that the subject only uses 58 prospective high school mathematics teachers and only provides two assignments from the area between two curves. Therefore, it is necessary to do further research to study further what if more subjects are taken, and the material provided is more varied and more difficult. In addition, from the types of processes and products the mathematical connections made by the subjects still varied, some of which were still in the category of weak mathematical connections, and these failed to solve the problem. From these results, it is recommended that lecturers need to train students more intensely to solve Integral Calculus problems so all students can develop their mathematical connection abilities into a very strong category. To other researchers, it is recommended to develop a rubric to measure the level of the strength of mathematical connections that is more varied, for example how the level of mathematical connections with categories is very strong, strong, moderate, weak, and very weak.

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