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The Effects of Mathematical Modelling in Mathematics Teaching of Linear, Quadratic and Logarithmic Functions

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Abstract: This study aims to acquaint high school students with the process of modelling in mathematics teaching. The research lasted 5 weeks with a group of (N=36) high school students of Zenica-Doboj Canton (Bosnia and Herzegovina). Students had an opportunity to learn about functions and their properties, and subsequently about mathematical modelling with linear, quadratic, and logarithmic functions. Examples in the research were related to real-world phenomena and processes. The problems were composed of the following subtasks: creating or testing a model, explaining the results, finding the domain and range, and critical thinking about the model. The research identifies the importance of mathematical modelling in teaching. The results display a positive impact of such an approach on students, their thinking, attitude towards teaching, understanding of the materials, motivation and examination scores. The experiences that both students and teachers may have in a mathematical modelling framework could be extremely important for the academic success. A control group of 36 students took the final exam as well. The students of the experimental group got much better results than the students of the control group. Indeed, learning through mathematical modelling has been shown to contribute to all the aspects of students' expected development.

Keywords: *Applied mathematics, critical thinking, mathematics education, mathematical modelling, modelling.*

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Introduction

In addition to good understanding of mathematics, teaching mathematics requires the ability to transfer the knowledge. Long-term trends in mathematics teaching accentuate solving mathematical problems through real-life models. Mathematical modelling is a process in which students put in the acquired expertise to the unknown situations. As identified by Blum (1993), there is "a substantial gap between the forefront of research and development in mathematics education, on the one hand, and the mainstream of mathematics instruction, on the other" (p. 7). This gap most probably still exists, in many regions. The task of modelling is to encourage learner's ideas, expand the accumulated knowledge and facilitate comprehension of mathematical essence. Modelling is more than just addressing word problems. As stated by Marković (2011), "the problem-solving process continues with the selection of suitable methods and procedures in mathematics, where certain results are obtained and which must be translated into the real world, to be presented in terms of the original situation" (p. 36). The author further explains that in the paper, the one who solves a problem also confirms the validity of the mathematical model. Raising a generation that uses the acquired knowledge and skills in the right way should be the goal of every education system. Therefore, one must be ready for the challenges of the modern world. In that regard, education system plays a major role, preparing population which will push the society forward and lead it into a brighter future. Without quality education, there is no progress. Mathematical modelling undoubtedly develops the mathematical competencies and skills that affect the personal progress of students, who through the process of education should grow into critically oriented citizens. The way in which the process of learning through modelling influences thinking is unique and special. As always, the questions asked are crucial. The society of mathematicians could ask the following: "How much does the modelling affect students' motivation and success? What is the right way to inspire teachers to start using modelling in class? Is there enough education freedom for independent initiatives? How to find

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appropriate literature to cover all the high school fields of expertise? Which grading system will match the modelling technique? To what extent does the educational system support the implementation of mathematical modelling in schools?"

The development of mathematics and mathematical modelling has flowed in parallel throughout history. Despite this, mathematical modelling has only recently appeared in high school curricula in mathematics teaching, both in the world and in our country. Demands for a change in the current teaching practice are mainly due to business and economic pressure. Namely, practice shows that it is difficult to transfer school knowledge to real problems. It has been observed that the school prepares students with very extensive mathematical tools, but does not prepare them to use those tools. Therefore, there are requirements for students to be real "problem solvers" and to get ready for the upcoming challenges.

In 2000 BC, the civilizations of Egypt, India, and Babylon possessed mathematical knowledge and used some mathematical models to make everyday life easier. From Thales and calculating the height of the pyramid, Euclid and his "Elements", Eratosthenes and the diameter of the Earth, Ptolemy and the model of the solar system, Al-Khwarizmi and the algorithms, Fibonacci and "The Book of Calculation", all the way to Einstein and the theory of relativity, this has been a difficult but an important path in the development of human thought.

The authors of Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) report (Garfunkel et al., 2016) claim that "mathematical modelling can be used to motivate curricular requirements and can highlight the importance and relevance of mathematics in answering important questions" (p. 8). Students of all ages need to be as familiar as possible with the problem situations that "promote" the importance of mathematical ideas, and not just apply the rules they have previously learned in the class. As the domestic methodology of teaching mathematics is lacking in the literature on the topic of modelling, this study contributes to the promotion of modelling in the teaching of mathematics in Bosnia and Herzegovina. A considerable number of papers and studies in the world show the positive characteristics of the modelling process, but modelling is still not fully integrated into the teaching of mathematics and there is still a gap between the traditional school and such a teaching form. Mathematical descriptions of the basic principles of physics and nature have become the main driving force for modelling and developing mathematical theories. Over time, there has been a general belief that society as well as nature can be analyzed. Today, we can see that this is widely applied. Indeed, it has been shown that society functions according to laws that can be described in mathematical form. Somewhat later, models appeared in the field of economics, and today we could say, without exaggeration, almost everywhere. Modelling real problems, and especially mathematical models, have been so important to the development of mankind that they have independently developed similar methods in China, India, and Islamic countries. Niss et al. (2007) state that "the modelling is not a part of a traditional teaching but represents a modern and quality teaching mathematics". Indeed, the school focus should be on the science applications as the gate to the problems of the outside world, so that students would surely receive and process the knowledge, be more interested and intrigued by what they learn, and perhaps then the questions such as "What will this do for us in life?" would not come up that much.

There are many theories about the definitions and explanations of mathematical modelling. Garfunkel et al. (2016) in GAIMME report define it as a "process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (p. 8).

Contribution of this paper to the literature

- The aim of this research was to examine how mathematical modelling in mathematics teaching affects student performance, efficiency, and motivation.
- The experiment reveals important progress of the group in understanding the mathematical concepts such as function, function properties, graph, domain and range, value, etc.
- Learning mathematical modelling gives significant results with all students, regardless of the previous knowledge, and it shows how much modelling influences students' critical thinking skills.

Research Problems and Questions

Lately, there have been numerous discussions about the education reform. We can hear that a "school must be more open to the labor market, the system has to look into the requirements of the society and focus their interest in that direction, etc." Unfortunately, not enough has been done yet to bring teaching closer to the real-world problems. In addition to all the above, the application of technologies in teaching is crucial. This study shows how valuable it is to know how to deal with a software such as GeoGebra, Wolfram Mathematica or some other graphical tool. Students are much closer to the digital world today than they have ever been, therefore, it is important to emphasize the value of educational software for learning and teaching. Through the research, the students did not hide their satisfaction with the way mathematical modelling works. They say that to them, although short, it was a wonderful, refreshing, promising and inspiring experience. Researchers noticed a deficiency in the students' critical thinking abilities. Therefore, mathematical modelling is the right aid for that problem. Modelling tasks should be constructed in a way that leaves enough space for students' suggestions, opinions and criticisms. How can we expect from those who do not learn today about the importance of their own views to have an opinion about the ongoing issues tomorrow? Furthermore, the lack of teaching skills within the application of modelling in teaching is something we should work on. Analogous to Marković's (2011) study, to verify the teacher's issues with the application of modelling, researchers asked 25 high school teachers the following questions:

1. Does the curriculum contain mathematical modelling topics?
2. Do you use mathematical modelling in preparing lessons?
3. From your perspective, is it difficult to implement mathematical modelling in current curricula?
4. Is there a space in the curriculum to add more mathematical modelling?
5. Can mathematical modelling provide better understanding of the essence and application of mathematics in other scientific fields?
6. Would mastering the content by mathematical modelling attain greater level of student motivation?

Answers revealed that mathematical modelling is still in its infancy in school habitat. The study confirms that mathematical modelling is not sufficiently represented in the curricula. An important research was conducted by Zeytun et al. (2017), with 19 prospective teachers who entered a mathematical modelling course designed for PTs at the public university in Turkey. One of the study goals was to learn modelling with the focus to results. The conclusion was that "When faced with difficulty at a point in the solution process, the ambitious teachers sometimes ignored the most relevant or main variables, or made assumptions that supported their intuitive answers to be able to reach a solution; in other cases, they considered their solutions as valid, with no need for further verification." (Zeytun et al., 2017, p. 2)

Even in Indonesia studies are done into this matter. As Vitoria et al. (2021) present, there are not many publications in which influence of modelling on students' achievement is shown. At this moment, almost everything depends on a teacher, and the will for self-improvement and personal growth. The key is the teacher's sense of placing students in a quality learning environment. The methods used are quite conventional, it is up to the lecturer, as an individual, to work on and learn about the modern principles of teaching, as they do across the world. The lecturer should not be exclusively dedicated to collecting and processing facts but to developing new ideas and opinions in students. The ultimate goal of education must be a symbiosis of knowledge and skills, as a preparation for the challenges of the future. Hence, the research problem was to reveal underrepresentation of modelling in schools and to show its effects to students' critical thinking and educational interest. Therefore, this study produced the leading question:

"What is the effectiveness of mathematical modelling on improving critical thinking skills, academic motivation and success among high school students?"

From there on, specific sub-questions raised:

1) What is the effect of modelling toward students' achievements?
2) How does modelling affect the understanding of a function and its properties?
3) Does mathematical modelling have impact on students' motivation?
4) Is the mathematical modelling approach more productive than the conventional way?

Research Objectives and Tasks

As this topic is not very relevant in our schools, one of the main goals was to encourage teachers to start using mathematical modelling in the classroom. Unfortunately, mathematical modelling has not yet taken hold in our country (Bosnia and Herzegovina). There are many problems that contribute to this: the discrepancy between the curricula and the modern learning outcomes; lack of literature about mathematical modelling in the native language; insufficient training of teachers and students etc. That is why this research was conducted, to examine this issue and to do something significant for the educational community. The progress space prompted us to make the first step. In fact, a 5-week program of mathematical modelling was designed, which consisted of the following: learning about functions and their properties, using GeoGebra and Wolfram Mathematica for graphs, doing tests, studying mathematical modelling, examining functions with mathematical modelling, taking surveys, solving quizzes, answering questionnaires, playing quick games.

As the first objective, Google Classroom was created for each of the 3 groups separately (high school classes 1,2,3), with the total number of 36 participants. As a prerequisite for mathematical modelling is knowledge in the field of functions, it was necessary to address that issue. Furthermore, it was necessary to get the students acquainted theoretically with the concept and significance of mathematical modelling. The students did not encounter this way of working before, so the introductory part had to be more thorough. Students and teachers need to be motivated to use modelling. Indeed, teachers will be motivated to organize their teaching and to provide appropriate literature. Student's motivation lies in preparation for such a learning mode and the challenges of mathematical modelling itself. In fact, it is extremely important to examine both teachers' and students' attitudes and opinions before incorporating mathematical modelling into mathematics curricula. One of the main objectives of the research was to organize lessons based on mathematical modelling and familiarize students with a different approach, significantly away from the conventional one. In addition, software tools GeoGebra and Wolfram Mathematica were offered to students, to visualize problems and get to the solutions easier. With the help of these applications, it is possible to enhance the understanding of the connection between mathematics and real situations and problems. The booklet *Mathematical Modelling in Mathematics Teaching for High School Teachers* will remain as a product of this research, similar as Riyanto et al. (2019), who produced "modelling exercises, lesson plan, and student worksheets for senior high school mathematics modelling" (p. 12). Besides the theoretical part of modelling, the manual contains about 50 examples solved in detail (each made of 4-10 subtasks) and another 50 simple problems with shorter solutions. The following functions are covered with this paper: polynomial, linear, quadratic, exponential, logarithmic, trigonometric, rational, irrational and functions in two or more variables. It should be a great addition to the high school textbook literature. Everything that teachers need in teaching mathematical modelling will be contained there. Based on the defined problem and objectives of the research, there are the following tasks: check the teachers' obstacles regarding mathematical modelling in classroom, determine the progress of students' knowledge in the field of function theory, confirm that students of the experimental group show better results in mathematical modelling than the control group students (who did not take part in the program).

Research Hypotheses

H_1: There is a statistically significant difference between the results of the pre-test and the post-test (examination about function, its properties and applied problems).

H_2: There is a statistically significant difference between the results of the experimental group (who studied by mathematical modelling) and the control group (who studied using the conventional method) at the main test.

Research Significance

The significance of this study is a serious approach to the problem of mathematical modelling in high school. Not many studies were conducted in Bosnia and Herzegovina, with the exception of the studies by Marković (2011) and Zlokapa (2012). Namely, mathematical modelling is still non-existent in high school mathematics curricula of the Zenica-Doboj Canton. The steps of the research are planned and realized in detail through the following: the opinions of students and teachers, examining motivation and success through modelling, knowledge acquired through modelling and developing critical thinking skills. All this has led to a single conclusion, mathematical modelling significantly affects students' efficiency and interest in learning and research. A request was sent to the domestic *Committee for Reform and Improvement of the Curriculum* to add mathematical modelling topics to the innovated curricula. That will be the stepping stone to a rapid application of mathematical modelling in high schools of Zenica-Doboj Canton. Researchers are positive about this request, hopefully the answer of the Committee will be affirmative. Enriching the current literature of high school mathematics is the current leading objective, the manual/book is being prepared and will be published soon. This study should encourage schools and universities to use mathematical modelling more in the future. The results of the study will be valuable for lecturers in teaching mathematics. This might be a great opportunity for the teachers to enhance mathematical subjects. Likewise, the results will be beneficial for the learners because they will inspire society to carry out the active learning through mathematical modelling process. It is still important to emphasize the critical thinking of students as a result of work through mathematical modelling.

The presented research is designed not only to present the most important works, results, literature in theory of mathematical modelling, but the practical solutions as well. In addition, to point out the importance of mathematical modelling in local sense (Bosnia and Herzegovina) as emphasized by the work of Vitoria et al. (2021) related to the shortcomings and problems of this practice in Indonesia, it is worth mentioning that research can be taken as a good basis for the analysis worldwide. The findings of this study were that the students, even didn't encounter modelling earlier, were thrilled and enjoyed learning mathematics through the mathematical modelling. Similar results are shown in Riyanto et al. (2019), where it is further argued that mathematical modelling should be implemented in teaching of mathematics starting from primary school and building up to the faculty level. Further, it turned out that the students know the concepts of mathematics, but only seemingly. In function-related problems, it could be seen that students know the theoretical part but not how to apply this knowledge to the problems related to the real world. After getting acquainted with modelling, their reasoning is at much higher level, students not only know how to analyze and solve complex tasks, but also offer various solutions, explanations, suggestions and critiques. Although the participants were quality students, some of whom had already proven in math competitions, prior knowledge was not crucial in the process. Mathematical modelling has been shown effective for all students. For most of participants the topic was new, which is probably the reason for their exceptional interest and will to participate.

Literature Review

Modelling in mathematics teaching strives to develop a better understanding of mathematical concepts, teaches students to understand mathematical problems, to observe facts, to formulate and solve problems that are the result of specific real-life situations. Mathematical modelling is the process of obtaining a mathematical description of a phenomenon, such as physical or chemical phenomenon, process in the field of economics, biology, sociology, etc. At the same time, this description must be relatively simple and accurate enough to fit its purpose which is defined by the one who creates the model. Authors Vargas-Alejo and Cristóbal-Escalante (2014) in their study posed the following questions: "What kind of mathematics teachers are required? What kind of educational programs, projects, and processes should be developed? How should mathematics teachers be supported to incorporate activities into their teaching practices that motivate students to commit to learning? What kind of mathematical activities and learning processes do teachers emphasize in their mathematics instructional proposals when they have to include problems? How do teachers think students learn mathematics? How do teachers conceive their role and the role of their students during instruction?" (p. 1).

Vitoria et al. (2021) believe that "mathematical models are mathematical representations of real-life problems which are used to simplify the problems and also for future prediction. Here is the seven-step model (Blum & Leiβ, 2007).

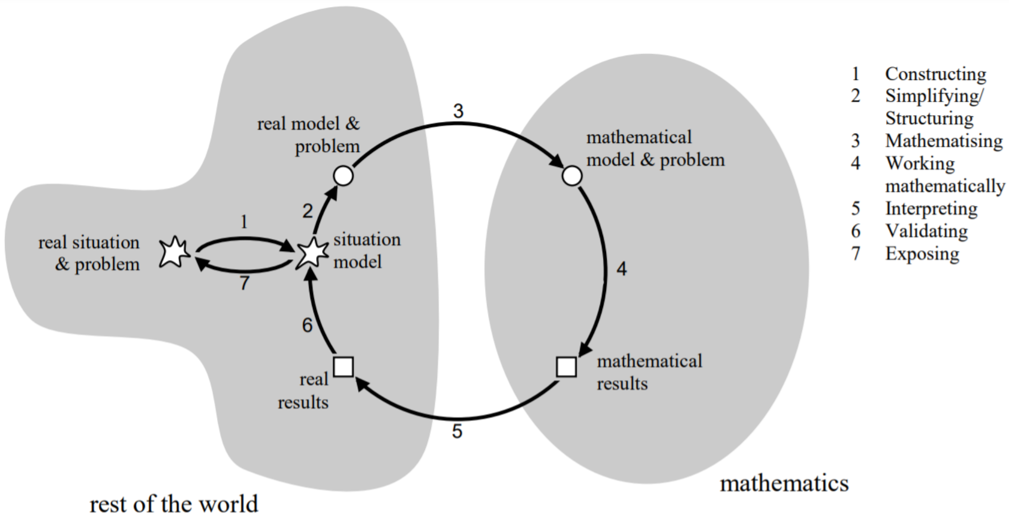

Figure 1. Modelling cycle by Blum and Leiβ (2007, p. 225)

It is a definitive cycle which starts with the real-life situation/problem, coming to model construction, model analysis, mathematical results and the discussion of what the results obtained speak for the physical problem given at opening. If the results are not satisfactory, certain steps through the modelling process could be revised and fixed. That way, the cycle continues until the moment it matches the set criteria.

The essence of the modelling process is to select only those properties of the observed process that represent the necessary and sufficient characteristics to describe the process accurately enough considering its purpose.

It is clear that the mathematical models go beyond the physical characteristics of real-life situations. Authors of GAIMME (Garfunkel et al., 2016) presented a simple method of transforming mathematics problem into the modelling problem (see Figure 2).

Figure 2. Illustration of a path from math problem to a modelling problem. (Garfunkel et al., 2016, p. 12)

Strict procedure from a mathematical problem to the modelling/real-life task is linked with intermediate steps. Introducing modelling has to come after students are familiar with the following: simple word problems, applied exercises and small modelling tasks.

At school, it is especially important to solve open-ended tasks and real-life problems. This is also mentioned by the respondents in the study conducted by Vos et al. (2019), which highlights the joy that can result from problem solving. The same authors insist that further research is needed that links the affective perspective to the design of mathematical modeling tasks, enjoyment and satisfaction should be reflected in new approaches to teaching mathematical modeling and are important for student learning, promoting mathematical development and student curiosity. The design of mathematical modelling is to express the mathematical problem in some context, in a mathematical way. It is a cyclical process in which we connect a mathematical model with a process or phenomenon. Many authors find mathematical modelling challenging because it presupposes knowledge of mathematical processes, tools, and techniques just as much as critical thinking. Such challenges can have multiple benefits in teaching and great potential to contribute to the development of critical thinking in students. Gloria Stillman (Kaiser et al., 2011) presents the modelling process with the following diagram (Figure 3).

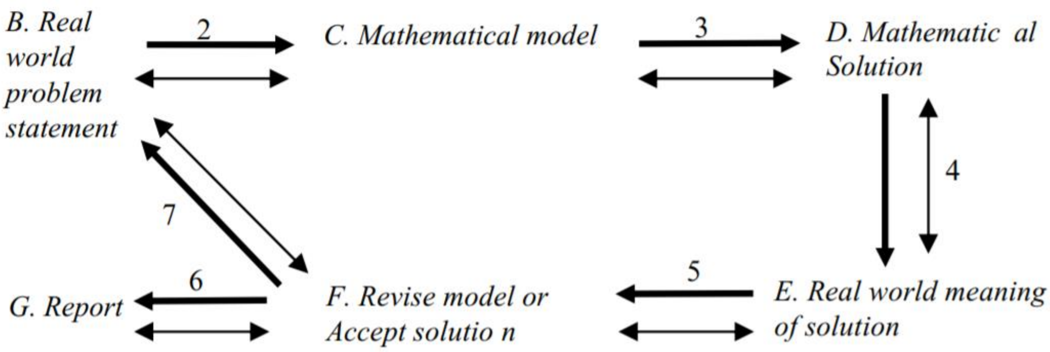

Figure 3. Modelling process by Gloria Stilman (Kaiser et al., 2011, p. 167)

This diagram could be amended so as to become a rounded process, where we go back to the model and its application or possible amendment. The main criteria for selecting modelling problems are their availability to students, i.e. the actual context of the example should be understandable without too much extra work. Possible approaches to modelling should be within the mathematical horizon of students. According to GAIMME report (Garfunkel et al., 2016) it's possible to find the modelling process diagram with all the components for the modern mathematics teaching. Figure 4 shows a profound representation of the mathematical modelling, something to be recommended to the math teachers all around the world.

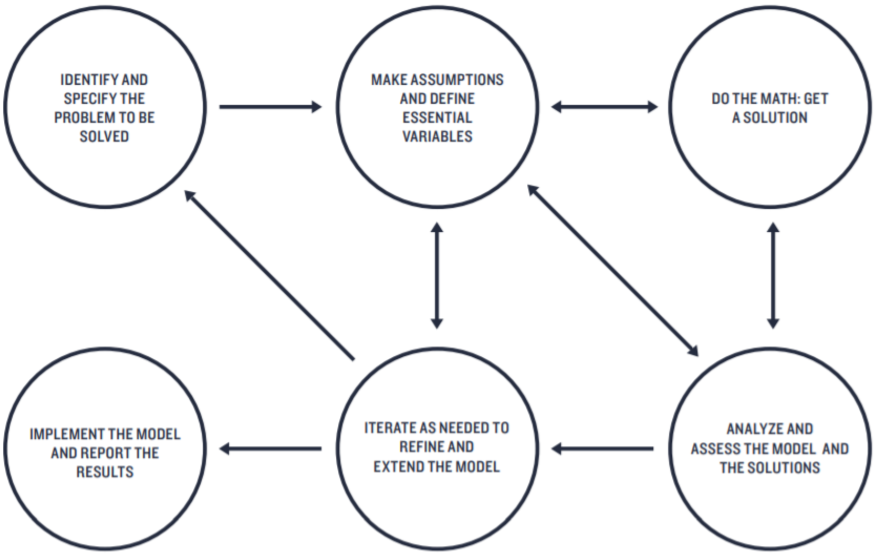

Figure 4. The math modelling process (Garfunkel et al., 2016, p. 13)

Frejd (2020) emphasizes the importance of analyzing the scientific potential of mathematics teachers with the aim of accumulating it, which could lead to a better practice of mathematical modeling in schools. Problems that we solve this way rarely appear in our teaching practice. Does that mean we don't do modelling? - No. The modelling tasks can be accessed very differently - the difference is shown in the objectives and the performance. Gusić (2011) believes that there are three basic ways of modelling:

- *Standard application* - students learn to do modelling and apply it in controlled conditions.
- *Direct modelling* - students solve a real problem by choosing, with the help and supervision of teachers, some of the learned models.
- *Open modelling* - students solve real problem independently.

The more connections between variables are known, the more accurate the model will be. Therefore, it is sometimes useful to incorporate selective information into a model based on intuition, experience, expert opinion, or the persuasiveness of a mathematical expression. The more variables and the relationship between them are covered by the model, the more complex the model is and its usage is less simple. More complex models are often more precise, so it is necessary to determine the desired accuracy of the model and thus choose the level of complexity. Wherever possible, simpler solutions should be used.

As stated in Zlokapa (2012) the curricula of some countries (such as the Netherlands, Australia, Germany), include mandatory modelling in a high school program. Standard small models apply well in schools and fit well with conventional curricula, in our country (former Yugoslavia) that is not the case. Modelling has recently begun to be applied at the university level, and most of the modelling is done by the mathematics education experts (Zlokapa, 2012). The education system is supposed to work under the strict assumption that mathematics is all around us. The challenge is to recognize its presence, join it in the right way and productively apply it where possible. The goal of modelling according to Galbraith and Holton (2018) should: "develop a systematic and efficient approach to solve individual problems perched in real life, and enable the effective solution of the challenges posed by the modern world" (p. 6).

Unfortunately, modelled tasks are still not significantly represented in mathematics teaching, as they should be. The steps of mathematical modelling could be divided into the ones before and those after the active introduction of mathematical modelling in teaching.

The first part could consist of:

1. analysis of mathematics curriculum,
2. supplementing the curriculum with the areas of mathematical modelling,
3. finding or creating appropriate literature,
4. teachers' education to work in the framework of modelling.

The second part would refer to teaching, i.e. modelling classes. Galbraith and Holton (2018) presented the steps in the process with a diagram in Figure 5, for work within modelling frameworks.

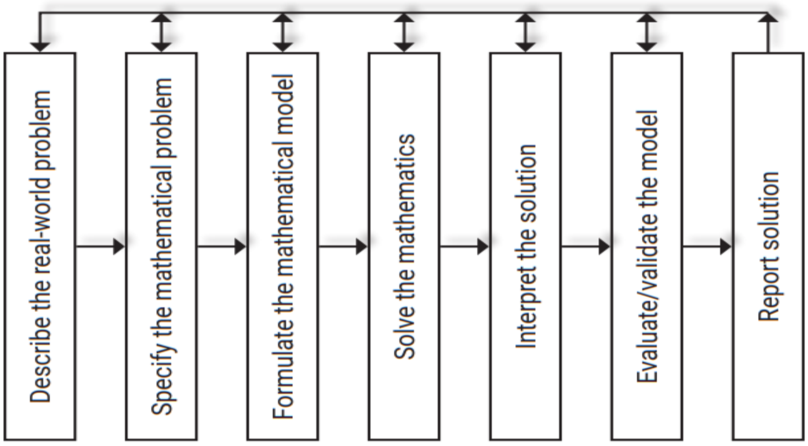

Figure 5. Mathematical modelling framework (Galbraith & Holton, 2018, p. 6)

The structure of a modelling process always begins with and comes back to the issue of real-life phenomena, but the closure is reserved for the report. However, if the report completes the norms and principles the case is closed, otherwise it is mandatory to go back and revise the previous results.

Apatić (2016) lists out four approaches to mathematical modelling:

- *Empirical approach* in teaching mathematics is used to find the relationship between quantities without already giving relevant empirical data. In this approach, the data is often given in the form of a list, table or points in the coordinate system;
- *Theoretical approach* is used when, based on assumptions, it is possible to connect the described situation with the mathematical concepts and simply answer the questions of interest;
- *Simulation approach* is the one in which, based on the given assumptions, a part of the phenomenon is described, but not the phenomenon itself;

- *Dimensional approach* is stated as a dimensional analysis and refers to the establishment of relationships between values with a respect to units of measurement.

From the 1990s onwards, it became clear that the word problems and even application problems are insufficient in influence the students' academic quality. By solving modelling tasks students have to: investigate, collect data, analyze, create models, apply the model to a real-life problem, and produce ideas. Blum (1993) emphasizes four "essential arguments for the modelling usage, mainly based on general objectives in mathematics teaching: pragmatic arguments (benefit students to deal with the real-life situations); formative arguments (acquire general skills and attitudes); culture arguments (promotion of mathematics as a part of human history and art), psychological arguments (help deeper understanding of mathematical topics)" (p. 5).

There are many programs used in teaching mathematics today, e.g., GeoGebra, Desmos Graph, Wolfram Mathematica, etc. The modelling process does not depend on the software but can accelerate and enhance student development. Modelling and digital technology are powerful tools in teaching and exploring mathematics.

"Unfortunately, although many educational systems have made a significant progress in this area, there is still no indication of such a reform in our state. Either way, modelling is a task that requires responding to challenges for both students and teachers because of the multiple processes it entails within the classroom. Particularly complex are cases involving the integration of digital technologies and curricular programs" (Molina-Toro et al., 2019, p. 4).

As stated by Hernández et al. (2016, p. 339) "modelling in teaching is a challenge, for several reasons, mostly for the new teachers". Modelling is presented by the illustration in Figure 6 (Carlson et al., 2016; cited in Hernández et al., 2016). In the core of this system is a teacher, as the one who initiates and maintains the system.

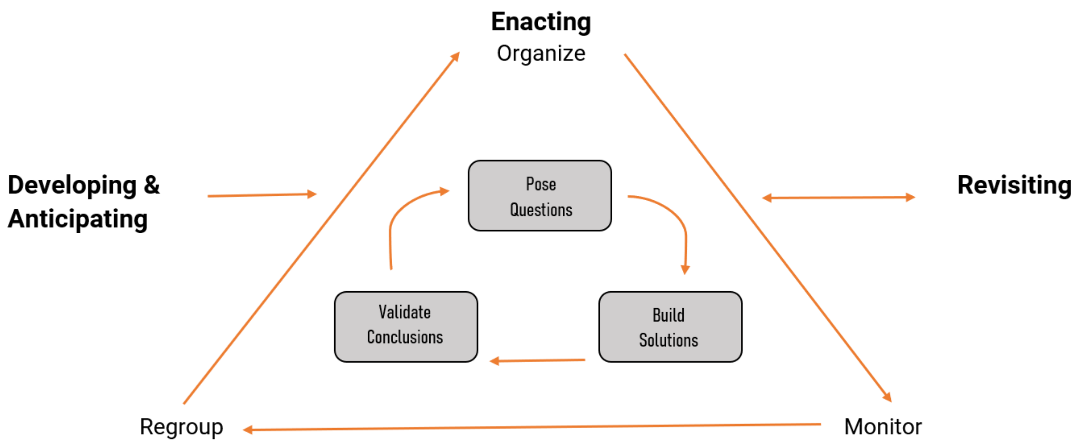

Figure 6. The framework for teachers' roles in modelling (Carlson et al., 2016, p. 122).

Teachers could ask the following questions: "What questions will students have about the context? What additional information will they need or want? How will they get that additional information? What assumptions will they take when they start building their model? How can they help students feel comfortable in setting hypotheses? What problem-solving strategies are students most likely to use?" (Hernández et al., 2016, p. 339)

Drijvers (2013) sets criteria for involving digital technologies in mathematical modelling. Suggestions are posted as three phases: studies that contribute to the field, integration of technology in mathematics education, and pedagogical functionality of the digital technology. On the other hand, Molina-Toro et al. (2019) state that "technology is used as a resource of real-life investigations, and as instrument for reorganizing the production processes of mathematical knowledge" (p. 2). These attitudes are expected because technology will never be able to do everything for us, it is man who creates new ideas. For simpler modelling, simpler software such as GeoGebra or Wolfram Mathematica can be used.

It would be ideal if modelling tasks were linked to the local companies, schools, colleges, businesses, successful athletes, actors, etc. In that way, a better effect in terms of student motivation would certainly be achieved. However, if teachers are not able to create their own assignments there are many materials that can be fetched for free. The work of Rosa et al. (2020) discusses the use of ethnomodelling in the mathematics curriculum. "In this context, ethnomodelling has emerged as a pedagogical action that relates to the research fields of ethnomathematics and mathematical modelling. Therefore, modelling provides a valuable pedagogical approach appropriate to an ethnomathematics programme because it contextualises mathematical knowledge developed locally" (Rosa et al., 2020, p. 107).

Some authors point out (e.g., Barbarossa in various works), to confirm whether a model is good, students should work in pairs or groups and finally discuss the model they have created. On the other hand, Vargas-Alejo and Cristóbal-Escalante (2014) suggest that "students had to gain mathematical knowledge before solving mathematical modelling tasks". It is indispensable to check out the mathematical modelling studies done so far, as done in Turkey, for instance (Çelik, 2017).

This research presents that, in the period between 2004 and 2015, only 8 out of 49 approved papers, or 16.3% of all the studies, were related with the secondary school. The main question is “Where to start to improve?” Most probably at the university level, students who are to become teachers one day, should be prepared for such a teaching method. As Zlokapa (2012) stated, modelling is underrepresented in schools (except in some large countries, although it is not represented there at the level it should be). In developed countries, modelling is applied in high schools and university institutions. Zlokapa’s opinion is that modelling should be applied at the higher level of education, especially at universities, because students at the lowest level do not have the knowledge needed to create a model. The same author believes that modelling requires the possession of a wide fund of knowledge and the ability to think creatively. Students of the Bachelor’s program Applied Mathematics of the Eindhoven University of Technology, were assigned a simple three-step representation of the modelling cycle. That study was performed by Perrenet and Zwaneveld (2012), which could be a starting point for a more quality modelling integration into the educational system.

In the field of mathematical modeling competence, research is still to be explored. A review of studies by Cevikbas et al. (2021) shows that geographical origin is of exceptional importance, the research of the mentioned competencies is particularly produced in Europe, while in other parts of the world researches are expertise-based. Most of the authors of mathematical modeling competencies come from Europe, then from Asia, Africa and America, and only one comes from Australia, the most prominent country in this field is Germany. Cevikbas et al. (2021) also state that in about 32% of studies the number of participants in the experiment is less than 50, and 45% of those studies in which high school students are included.

Authors Garfunkel et al. (2016) in GAIMME propose a template on how teachers could trigger the mathematical modelling in the classroom. Suggestion by the same authors is to take baby steps at the beginning, consult the ones who were already involved in the process, do small modelling problems and later full modelling exercises, and finally share ideas with the school administration.

As reported by Garfunkel et al. (2016) in GAIMME, here is a list of platforms where teachers could find the needed modelling materials:

- COMAP (www.comap.com),
- SIAM/M^3 (m3challenge.siam.org).

There is another delightful project, named IM^2 (www.immchallenge.org.au/) “International mathematical modelling challenge organized by the Australian Council for Educational Research (ACER- www.acer.org/au/)”

These materials are available and can be downloaded for free. In addition to the above sources, there are many textbooks used in high school teaching around the world and with the entire chapters of mathematical modelling tasks. Problems and tasks solved through this study were taken, partially or completely modified and presented to students.

The following textbooks are the most dominant as a literature of the experimental part of this research:

∇ *Precalculus* - Abramson, J. P., Falduto, V., Gross, R., Lippman, D., Norwood, R., Rasmussen, M., et al. (2014),

∇ *Algebra 1* – Larson R. & Boswell, L. (2014),

∇ *Precalculus-An Investigation of Functions - Edition 2.2* - Lippman, D. & Rasmussen, M. (2020),

∇ *Modelling, Functions, and Graphs - Algebra for College Students, Fifth Edition* - Yoshiwara, K., & Yoshiwara, B. (2018).

Methodology

Research Design

The research aim was to examine the effect of mathematical modelling in teaching mathematics to improve student thinking skills and academic motivation. For such a purpose two groups were created: the experimental group and the control group. A mixed methods design was used with its quantitative and qualitative methods. For instance, questionnaires were used to examine teachers' opinions about modelling and exams/quizzes to check the acquired knowledge. This research is pragmatic with a focus on mathematical modelling in class as a tool to improve the mathematics teaching, better understanding of difficult mathematical problems and students’ academic motivation. By processing the topic, in order to confirm or reject the hypotheses, the following methods were used: *method of analysis, survey method, comparison method, experimental method and statistical method.*

Sample and Data Collection

Strategy used for this study is similar to the one conducted by Blum and Borromeo Ferri (2009), with the *treatment, pre-test, post-test and follow-up (main) test*, which was a research with more than 600 students. In addition, alike a study of Riyanto et al. (2019) this research composed of 3 steps: analysis, design, and evaluation. A sample of the study population was deliberately selected consisting of 36 high school students, in the second semester of academic year 2020/2021, in the Zenica-Doboj Canton. Each of the high school grades, first, second and third, had 12 candidates within their group, who studied modelling with: linear, quadratic and logarithmic function, respectively. The experiment was organized "online" and conducted with: Graphic tablet (HUION), PDFAnnotator, Google Workspace (former G Suit), Zoom Meetings, GeoGebra, Wolfram Mathematica, Quizizz, MSOffice, WordWall. The program was designed as the set of: presentations, exercises, quizzes, tests and surveys, which lasted around 10 lessons (5 encounters), for each group separately. The program could briefly be described in five stages (meetings):

- INTRODUCTION PART: It was used to get acquainted with the experiment, using the PowerPoint presentation. At the end of the meeting, the students were given a PRE-TEST (17-20 short problems) using the online platform Quizizz. The test was aimed at examining students' knowledge of the concepts of function, its domain and range, value and argument, growth and decline, graph, etc. Everyone picked a code name for publishing scores.
- SECOND MEETING: The first three problems were solved; solutions were written with the students' assistance. One of the tasks of the exercise was left in Google Classroom for students to do independently, until the next meeting.
- THIRD MEETING: Likewise, the next three tasks got worked out. For the exercise, the students were given another task.
- FOURTH MEETING: The last active meeting was used for the POST-TEST, which was performed via Quizizz. Furthermore, students were solving the mathematical modelling exam, which consisted of the following problems: creating and testing model, sketching graphs, finding domain/range and a critical thinking question about the obtained model. Students were able to view the results later in the Google Classroom and file any complaints about the score. The lists are published with the encrypted names.
- FINAL PART: The final results were published, which are made up of the following: attendance, activity in class, practical exercises, quizzes and examinations. The most successful (3) students from each group were presented with the certificates and the proper awards (see Table 1).

Table 1. Performance of "logarithmic modelling" group

	maximum points	*20*	*10*	*10*	*10*	*20*	*30*	*100*
#	**nickname**	**attendance**	**activities**	**ex. 1**	**ex. 2**	**quiz**	**test**	**total**
1	mm1110	20	10	10	9	19	30	98
2	KS2003	20	10	8.5	10	18	29	95.5
3	Tuples4	20	10	10	10	14	29	93
4	Elektro2003	20	3	10	10	20	27	90
5	Abraham1221	20	6	10	8	18	28	90
6	Melankolik917	20	10	8		18	26	82
7	SnowQueen19	20	7	7.5	6	12	22	74.5
8	TheKubo21	20	3	10		16	24	73
9	Daisy2003	20	3	10	6	15	18	72
10	DrDis2002	20	10			15	25	70
11	Happy257	15	2	6	5	14	26	68
12	Nina3	20	3			8	22	53

Pre-test and post-test were conducted as online quizzes, using the Quizizz app (https://quizizz.com/). Table 2 presents a descriptive analysis of regulated tests and their values. The tests were the same type, consisting of 17 short questions, grouped into the categories:

- *Theoretical part (6 points)* - recognition of basic concepts related to functions;
- *Graphical solving (8 points)* - function graph, function value, domain and range;
- *Problem-solving tasks (6 points)* - word / challenging problems.

As the final phase of the program, the main test was executed, in which a control group of 36 (3 × 12) students was involved, for the comparison. Comparably to the program of students of "Bachelor's program Applied Mathematics of the Eindhoven University of Technology" (Perrenet & Zwaneveld, 2012), the modelling test (MT) carried 30 points and consisted of the following subtasks:

- *Creating model (7 points)* - based on the given data, create the model;
- *Graph (4 points)* - drawing graphs, recommendations: GeoGebra, Wolfram Mathematica;
- *Investigating model (12 points)* - test the model through the given questions;
- *Domain and Range (4 points)* – determine the set of definition and the set of values;

- *Critical thinking (3 points)* **-** review of the model quality, students' opinion on the obtained results, connection with everyday phenomena and processes.

Students displayed extremely positive attitudes about mathematical modelling in mathematics teaching, the approach that this process carries has been confirmed as positive, innovative and refreshing for most participants.

During the experimental program, students were subjected to the initial testing in the form of pre-test (PRT), and testing at the end of the program in the form of post-test (PST). The main modelling test (MT) is applied after performing the experimental program. Additionally, a control group of each class is formed for the purpose of MT.

The authors of this study mutually designed the tests. To ensure the safety of the scores, the tests were graded by five mathematics teachers. The objectivity of the tests was ensured by placing the students in a similar testing condition, while the examiners followed clear and detailed instructions in assessment check. The correlation (Pearson's correlation coefficient) between different test evaluators is very high: PRT ($r = 0.96$, $p < 0.01$), PST ($r = 0.98$, $p < 0.01$) and MT ($r = 0.99$, $p < 0.01$). After discriminant analysis for each of the 17 pre-test and post-test tasks, as well as 5 main test tasks individually, the reliability of the instruments is determined by calculating the Cronbach's coefficient for PRT ($\alpha = 0.85$), PST ($\alpha = 0.83$), and MT ($\alpha = 0.82$).

Data Analysis

For the purposes of testing and data processing, basic (descriptive) parameters were used, such as arithmetic mean and standard deviation. To examine the existence of statistically significant differences in the results, the ANOVA test was applied. The t-test was used to analyze paired samples to prove statistically significant differences between the groups before and after the experiment. All the analyzes were performed in the SPSS-26 statistical program. Scores of the pre-test are approximately normally distributed for all groups, with a skewness of 0.155 (SE=0.393) and kurtosis of -0.640 (SE=0.768). This evaluation is confirmed by Shapiro-Wilk's test with the value of $p = 0.328 > 0.05$, as well as visual inspection of histogram. All the above lead us to use parametric tests, because we obtained similar results for other tests used in this paper. There were similar results for the other two tests.

Findings / Results

Based on the sample ($N = 36$), it has been proven that there is a statistically significant difference between the pre-test and post-test results. The mean value of the pre-test among all students (36) was $X = 13.27$, and standard deviation $SD = \pm 2.854$. Both pre-test and post-test questions and problems were related to functions and their properties. By questions contained in the *Theoretical Part* students' progress was examined in respect to their knowledge about and perception of the basic function elements. Further, the *Graphical Part* was about the graphs recognition and to check the values out of the given graph. Additionally, *Problem-solving* tasks were the most challenging, but at the same time, students demonstrated the greatest breakthrough coming from a score 56.9% to a stunning mean value of 80.6%.

Table 2. Students' Performance at Pre-test and Post-test

	Theoretical part	**Graphical solving**	**Problem-solving tasks**
Pre-test	71.8%	69.4%	56.9%
Post-test	83.3%	81.6%	80.6%

At the post-test, the mean value of $X = 16.36$ and deviation $SD = \pm 3.015$ was obtained. The greatest progress is noticed in *Problem solving*, i.e., *understanding mathematical concepts and application to word problems*. It is undeniable that the experiment/program affected the students' knowledge quality in all the fields relevant to the functions.

Table 3. Relation of mean values, pre-test and post-test

	$\bar{x}$	**N**	**SD**
Pre-test	13.28	36	2.854
Post-test	16.36	36	3.015

The analysis of the t-test displayed that there was a statistically significant difference between the *Pre-test* and *Post-test* ($p < 0.05$), [$t = -6.485$; $p < 0.001$].

Despite a small sample of 36 participants, the difference in knowledge about functions before and after the experiment is huge, which is shown by coefficient of significance ($p < 0.001$), and implies the remarkable impact of mathematical modelling on a deeper understanding of mathematical concepts which brings the quality of student's academic development.

The next phase of the research was designed to analyze the difference between the experimental and the control group within the already mentioned exam fields.

Table 5. Differences between the results of the experimental and the control group

Exam parts	Group	N	x̄	SD	t-test
Creating model	Exp.	36	6.3611	1.35547	t=4.437, p<0.001
	Cont.	36	4.3889	2.29630	
Graph	Exp.	36	2.4722	1.62983	t=4.438, p=0.053
	Cont.	36	1.7778	1.35459	
Investigating model	Exp.	36	10.4444	2.62346	t=3.267, p=0.002
	Cont.	36	7.3333	5.07656	
Domain and Range	Exp.	36	2.1111	1.66952	t=3.273, p=0.002
	Cont.	36	0.9722	1.25325	
Critical thinking	Exp.	36	2.2222	0.95950	t=3.789, p<0.001
	Cont.	36	1.2500	1.20416	

According to the total tested sample of students (N = 36), and analysis of differences of mean values, t-student test of independent samples, it is noticed that there are statistically significant differences ($p < 0.05$) between the experimental and control groups in all the test parts of the main exam, except the *Graph* subtask ($p = 0.053$), which is also close to the significance limit. It is possible that the result is such because the students were able to use any tool for drawing graphics but the low number of points (4) provided for this section. The students who did not join the study program (control group) had a lower in score than the ones who joined the experiment, in all the subtasks. Mathematical modelling environment boosted the ability of the students to manage the real-life mathematical problems at a more profound level. The students engaged and advanced at critical thinking. The feedback is extremely positive and encouraging, participants are aware of how much mathematics and its applications are valuable to the personal prosperity of the individual. Every chance to show how mathematics teaching is important should be used, and one of the most graceful representatives is the teaching method called mathematical modelling.

Hypothesis H_1 was confirmed, there is a statistically significant difference in the results obtained by the pre-test and the test performed after the experimental program, about the properties of the functions. Likewise, hypothesis H_2 was proved as well. In addition to the results showing a significant difference between the control and experimental group in total, they demonstrated significant diversities in all the fields of modelling process (model creation, model investigation, graph of a function, domain and range, critical thinking about the advantages and disadvantages of the model). Based on the set hypotheses and the above results, application of mathematical modelling in mathematics teaching had positive effects in treating linear, quadratic and logarithmic functions.

Discussion

Frejd and Ärlebäck (2011) "have been investigating Swedish upper secondary students' mathematical modelling competency". Some aspects of this research were used for the ongoing study. The tests have been constructed to check the knowledge of the term function. Students of Brazil's public schools were engaged in a similar study (Rosa et al., 2020), with the aim to interpret the terms, signify mathematical concepts, and understand different points of view through the ethnomodelling. Results showed considerable effects on 38 participants, aged from 15 to 17, to the interpretation of function concepts, which led students to transcend the concepts of function presented in the local context.

Participants of the present study could win a maximum 20 points in both *pre-test* and *post-test*. At pre-test, the students presented the best success in the category of *Theoretical part* (71.8%), the weakest score at *Problem-solving tasks* (56.9% correct answers), in the part *Graphical solving* the percentage was 69.4%, while the average points throughout the total test points was 66.4 %. Contrastingly, the post-test scores were significantly better in each category, the percentage of success was over 80% (*Theoretical part* at 83.3%, *Graphical solving* at 81.6%, *Problem-solving tasks* 80.6%), namely 81.8% if we consider the entire post-test.

Regarding the experimental group, the best results were achieved in the area of *Creating model* (90.9%) and the weakest in the part of *Domain and Range* section (52.8%). Similar results were achieved with the control group. The students were the most successful in the field of *Creating model* (62.7%) and the least successful with the tasks of finding *Domain and Range* (24.3%). Apropos the whole test, the results of the experimental group with the average of 23.61 (out of 30) points are significantly ahead of the field compared to the results of the students of the control group who averaged 15.72 points.

The review of the modelling skills of Swedish high school students (Frejd & Ärlebäck, 2011) shows that "student's last mark and if they thought the problems in the modelling tests were easy or interesting, were factors positively affecting the students' modelling proficiency."

Students of this study (experimental group) scored 78.7% at main test, similar to the Indonesian students who scored average 78.82% (Sakinah et al., 2020), but more than Swedish upper secondary students (Frejd & Ärlebäck, 2011) with the average percentage of 55.57%. All the indicated studies have in common the significant progress of students' capability to deal with the real-life modelling processes, and hopefully one day to cope with the actual life problems and challenges. Problem-based learning is closely related to mathematical modelling, for primary school students it would be ideal to have problem-based learning as a groundwork for modelling. Continuing to use such modern tools would be the principle of mathematical modelling, with a good experience in tasks in problem-based teaching and learning. Teachers who do not have enough practice in modelling can start the modelling journey using problem-based learning. Studies that recently dealt with this problem are Bikić et al. (2016) and Sakinah et al. (2020). There are significant results on the impact of problem-based learning of mathematics as it was shown in these papers for the case of mathematical modelling.

Through the research students were able to validate their results using GeoGebra or Wolfram Mathematica software, as it was done by high school students in Germany (Hankeln, 2020), where it was shown the technology does not have too much impact on students' modeling competencies, which is clear for a simple reason, software cannot replace the work of human mind although it can be a corrector and a tool to confirm or accelerate the process.

The results show tremendous progress of participants, not only in better understanding of mathematical concepts but also in creating own opinion. Students were able to learn about mathematical concepts with a clear picture of the connection with reality, such learning is much more effective and lasting. Mathematical modeling concepts are plainer, more accessible and closer to the student. Unfortunately, in this field not much has been done in our educational system, school walls are still set high and much needs to be done to open school to the real world.

Conclusion

This research aimed at exploring the impact of mathematical modelling in teaching mathematics among secondary school students in the Zenica-Doboj Canton (Bosnia and Herzegovina), using the experimental approach. In addition, the paper strived to identify the most important advantages and challenges facing the application of the modelling strategy form among high school students. When analyzing the result, it became clear that the students of the experimental group exceeded the control group students in all the fields of expertise. The quantitative data present that the participants' perceptions of modelling were positive which finally led to better results. Furthermore, this study produced a *Manual for math teachers*, on mathematical modelling. It is essential that the teachers in this book will have enough information and tasks to independently start applying mathematical modelling in teaching. Steps to come in the process of modelling affirmation would be: education of teachers and students of educational faculties, competition organizations for students, workshops and symposiums for teachers with the necessary experience in this subject, discussions on modelling grading system, curricular upgrade with modelling field. With the mathematical modelling technique students have achieved undoubtedly superior results in the understanding and knowledge acquisition, were much more prosperous than those who were taught in a conventional, traditional way. Learning and teaching by mathematical modelling can greatly contribute to the development of students' opinions, increase motivation and interest in research, penetrate more into problems and their solutions, and develop critical thinking competence. The results obtained by solving tasks in an online environment lead to the conclusion that students did well in making them. Over and above, based on the obtained results, we can conclude that one of the basic tasks of teaching has been fulfilled. Students have been trained for independent learning as well as the application of the attained knowledge in solving tasks of everyday life.

As new generations of students increasingly use technology and digital content, the teaching process needs to be adapted to their environment and linked to the mathematical concepts, which are key to students' thinking design, in which modelling is one of the finest mechanism. This application of mathematical modelling with the use of digital tools demonstrates an innovative, creative and interactive way of acquiring and testing knowledge about linear, quadratic and logarithmic functions.

Recommendations

From the above analysis, a number of suggestions can be made about what to do in the future: (1) Trigger the modelling strategy within the university educational departments. (2) Work on university students' education in modelling competencies. (3) Hold training courses for the high school teachers to clarify the modelling procedures and its benefits on improving students' academic accomplishments. (4) Employ the programs with the group of high school students, perhaps in a form of competition. (5) Apply mathematical modelling in regular lessons. The current study suggests that a similar research could be conducted with a more numerus groups, and with a greater focus on the impact of mathematical modelling to students' motivation and the academic success. It would be useful to correlate the students' thoughts about modelling program and the scores at modelling examinations.

If the researchers decide to deal with the mathematical modelling investigations, the group of students can be larger (more than a 100), study can be spread to all the types of functions (exponential, trigonometric, polynomial, rational, irrational, etc.), all of that demands more effort but the results would be more quality and more reliable.

For the ones who plan to engage in a similar research or experiment, suggestion is to plan each step carefully and in detail. Due to the quality results, it is recommended to put focus to the single modelling field. At the very beginning of the research, it would be necessary to determine the level of students' modelling skills. Thereafter, set a clear criterion for evaluation of student's progress. The research could be conducted over a longer period of time (several months) where students would be given time and space to research and solve the exercise tasks. Before and after the experimental part, conduct surveys on applied tasks in mathematics teaching, mathematical modelling link between mathematics and real life, all this to look into how much impact does the mathematical modelling have on students' thinking. The research team could be wider to include a bigger team of students who will be joining the program. If the control group is not present, then before and after the experimental part, perform mathematical modelling tests, which should be around the same level. At the modelling main test, in addition to the problems of model creation, model testing and other model-related questions, it is crucial to leave space for students' creativity and critical thinking. It would be all-important if, in addition to the modelling task, the test contains the question "How do you feel about learning through mathematical modelling?" with the provided answer options: "Interesting", "Confusing", "Amazing", "Difficult", "Boring", "Useful", "Good", "Bad", etc. In such manner the students' main test results could be connected with their reaction about the modelling. Once the experimental part is complete the team of active teachers would form an outstanding crew for mathematical modelling training. This potentially great team would be able to work on appropriate customized literature that could be used in schools, with the problems related to a local or global characters, events, phenomena and processes.

Limitations

The research was limited to three groups of students in which mathematical modelling was studied through: linear, quadratic and logarithmic function. The study was implemented among 36 students (3 groups of 12) of secondary technical schools and gymnasiums of Zenica-Doboj Canton. The study results have been very useful for the research and could be significant for the future studies. Another group of 36 students made up the control group of this research, their results were compared with the results of experimental group (students involved in mathematical modelling program). Despite a small group of students, everything that was planned was achieved. Furthermore, a total of 25 high school math teachers deliver their opinions on general issues of mathematical modelling in school, through a survey. The program was executed through the March and April of the academic year 2020/2021. Lessons were organized online, via Zoom Meetings, and using: Google Workspace (Gmail, Drive, Classroom, Forms), GeoGebra, Wolfram Mathematica, MSOffice, WordWall, Quizizz, and for the writing solutions PDF Annotator and the Pen Tablet (Huion). Students mostly come from different schools so it was the only way to get them all in one place, virtual classroom. Besides the original intention of teaching by modelling, chance for improving online teaching skills was across the teachers/modelers, and "it was completed in the most professional and innovative way" according to students/participants. However, it was rather difficult to collect students and make them interested in the modelling program through the 5-6 "online" weeks. Luckily, from the very beginning students and researchers got along very well, so positive performance and exceptional results arose. A common agreement has been set, "mathematical modelling existence in math curricula would make students realize the true nature of mathematics science, and reveal the vital importance for the understanding of life manifestations."

Authorship Contribution Statement

Bikić: Idea, concept and design, methodology, drafting manuscript, data interpretation. Burgić: Admin, supervision, final approval, critical revision of manuscript. Kurtić: Technical details, experiment, data acquisition, data analysis, drafting manuscript.

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