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# **Effectiveness of Semiosis for Solving the Quadratic Equation**

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**Abstract:** The study examines the effectiveness of employing semiosis in the teaching and learning of the Quadratic Equation. The first goal is to compare results of De Saussure and Peirce models within the semiotic theory. The second goal is to determine the commonest effective semiotic objects student teachers mostly employ to solve for the roots in quadratic equations. This research method was mixed methods concurrent and adopted both quantitative and qualitative approach. The instruments for the study were teacher-made tests and interview guide structured on the likert scale. In the teacher-made tests, two sets of twenty questions were set and distributed to the respondents. The sets of questions were similar and each twenty questions were based on De Saussure and Peirce Semiotic Models. The analyses employed both quantitative and qualitative. In the quantitative analysis, three categorical *independent variables* were fixed on and Pierre and De Saussaure models, objects of Pierre and De Saussaure models, and diachronicity, trichronicity, categorization and quadratic equations, after satisfying normality and independent assumptions of t-test and ANOVA techniques. The qualitative analysis with ensured anonymity, confidentiality and privacy of respondents and transcribed responses from semi-structured interview guide. The results of the commonest semiotic objects improved significantly classroom interactions with Peirce model than with De Saussure model. They perceived the Peirce model as being broader, comprehensive, universal and ICT-compliant. We therefore recommended further quasi-experimental studies on semiotic objects to improve upon the use of cultural objects.

Keywords: De Saussure Model, effectiveness, Peirce Model, quadratic equation, semiosis.

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# Introduction

Research (Davis, 2013; Davis & Chaiklin, 2015; Presmeg et al., 2016; Roth, 2016) proposes various ways of drawing on social and cultural practices to scaffold deeper understanding of school concepts, make connections between school mathematics and everyday practices, enrich classroom discourses, develop appropriate chains of signifiers and signifieds (i.e. signs and symbols), provide systematic technical language for analyzing processes of mathematical thinking, symbolizing and communicating, and discover mathematical ideas through investigations. Semiosis is the study of activities with signs and symbols as applied in the general scope of signs, significations, socialization and representations (Presmeg et al., 2016); Radford, 2014;). Thus, studying the effectiveness of employing semiosis in solving for the roots of quadratic equations are continuously refined to fit the De Saussure and Peirce models.

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# Perspectives of Semiosis



Figure 1: Perspectives of Semiosis (Bartolini Bussi & Mariotti, 2008)

On Figure 1, the first perspective is attributed to the uses of artefacts for accomplishing mathematical tasks in social contexts (artefact-signs) and mathematical knowledge relevant to the uses of the artifacts (Bartolini Bussi & Mariotti, 2008; Maracci & Mariotti, 2009; Sabra, et al., 2014). This knowledge, expressed through the mathematical signs among complex relationships, is called the semiotic potential of the artefacts with respect to the given tasks. Thus, the uses of artefacts for accomplishing tasks afford teachers the opportunity to produce mathematics signs, orientate students towards intentional promotions of building relationships between the artefacts and mathematical knowledge.

Another perspective is attributed to making explicit distinctions between artefacts on personal and mathematical meanings (Mariotti & Maracci, 2009; Sabra et al., 2014). These distinctions depend on the concepts of knowing and knowledge, where knowing is the use of cognitive artefacts for accomplishing tasks and subsequently produce knowledge, and knowledge is the planned and gradual constructions of relationships towards promoting and expressing relationships among the artefacts and tasks. Thus, the semiotic potential of artefacts can be analyzed in both cognitive and epistemological domains in the design of semiotic models.

The third perspective is deployment of semiotic mediation in the classroom purposely for solving given mathematical tasks (Bartolini Bussi & Mariotti, 2008; Mariotti & Maracci, 2009; Roth, 2016). Thus, semiosis refers to the parts played by teachers in enhancing, selecting and shaping students' learning experiences, and helping students to move through layers of knowledge and understanding of the mediators (tools), the kinds of involvements and the changes of the symbolic tools-mediators. In other words, semiosis refers to the interplay of complex semiotic structures involving the mediator, content, mediate, circumstances, modality and location to bring desired goals.

The fourth perspective of semiosis is the art and science of creating human activities and developments that give consciousness, axioms and propositions on mathematical activities, where the principles of consciousness emerge from the uses of signs and artefacts to mediate mathematical tasks (Bartolini Bussi & Mariotti, 2016). That is, the conceptualization and reconceptualization of consciousness in developing universal scope rest on the following three major claims:

1. That signs are created and utilized in different classes of activities (home, work and leisure).

2. That individuals thinking is mediated by the same artifacts appropriate with the personal learning significances, situations and interactions.

3. That students progressively master socio-cultural resources and activities that mediate actions jointly undertaken with more experienced adults.

The fifth perspective semiosis is the conception of semiotic mediation and evolution of semiotic artifacts towards three activities:

1. Group activities to accomplish given tasks, where students work in pairs or small groups to produce common solutions from shared signs.

2. Individual production of activities with the artefacts in the eventual production of signs.

3. Collective discussions orchestrated by teachers' schemes (Bartolini Bussi & Mariotti, 2008; Maracci & Mariotti, 2009). These perspectives help teachers to design learning tasks, observe students' activities, collect and analyze written reports, manage classroom discussions provide semiotic models.

Comparing De Saussure and Peirce Semiotic Objects

# De Saussure Models of Semiosis

Ferdinand de Saussure (1857–1913) proposed a dualistic notion of signs, relating the *signifier* as the form of the word or phrase uttered to the *signified* as the mental concept. De Saussure posited that the sign is completely arbitrary, there is no necessary connection between the sign and its meaning, and there is no word inherently meaningful. Thus, it is rather only the signifier (i.e., the representation of something) that must be combined with the signified (i.e. the thing itself) in order to form a meaningful sign, and an empirical understanding of how humans synthesize physical stimuli into words and other abstract concepts from two elements---concepts and acoustic images. These notions differentiate De Saussure from Peirce thought in connecting the signifier and the object it signifies (Davis & Chaiklin, 2015; Presmeg et al., 2016).

However, De Saussure models have not been extensively researched in mathematics education to bring much impact. For instance, even though De Saussure's notions of synchronicity and diachronicity are ways of viewing both the sociohistorical and the socio-cultural processes in the production, where the synchrony is the cross-section and diachrony is the longitudinal section of signs and artefacts, the synchrony only involves the what is taught and learned in given situations while diachrony involves how ideas change during the processes of engagements over time with the signs and artefacts. This means both synchronic and diachronic notions inadequately explain the reasons behind the transformation of sign and artefacts into semiotic instructional models (Presmeg et al., 2016; Roth, 2015).

# Peirce Models of Semiosis

At the same time that De Saussure was formulating his model of the sign and laying the foundations of the structuralist methodology of semiology, a closely related theoretical work grounded on pragmatism by Charles Sanders Peirce was also formulated on the sign, semeiotic and taxonomy to a triadic model or trichotomic semiology, namely the representamen, the interpretant, and the interpretent/object (Lanir, 2019; Saenz-Ludlow & Kadunz, 2016; Van den Heuvel-Panhuizen et al., 2016).

The Peirce semiotic objects are the representamen (i.e. the form which the sign takes and not necessarily material), the interpretant (i.e. types of sign which is not an interpreter but rather the sense made of the sign) and the interpretent (i.e. something beyond the sign which refers to the interpretations of constituent relationships between signs and objects) to determine the effectiveness of signs and artefacts upon ontological and phenomenological consciousness (Bartolini Bussi & Mariotti, 2008; De Waal, 2013; Presmeg et al., 2016; Van den Heuvel-Panhuizen et al., 2016). Presmeg et al. (2016) subdivide the interpretants into intensional interpretants (i.e. determination of the minds of the utterers), effectual interpretants (i.e. determination of the minds of utterers and interpreters).

Peirce has also grouped the signs into three categories---icon, index and symbol. An icon stands for an object by resembling or imitating it in a visual way such as a map, picture, diagram or a graph to visually resemble some characteristics, an index refers to the sign which is the effect being produced by the object, and a symbol refers to an object by virtue of an established law, rule or convention. In other words, the symbol/symbolic is the mode in which the signifier does not resemble the signified but which is fundamentally arbitrary or purely conventional, and common examples are mathematical variables, mathematical coefficients, constant numbers, calculator modes, punctuation marks, number words, mathematics phrases and story problem sentences. The icon/iconic is the mode in which the signifier is perceived as resembling or imitating the signified such as portraits, 3D object, scale-models, graphs, diagrams and sketches. The index/indexical is the mode in which the signifier is not arbitrary but is directly connected to the signified such as degrees, powers, real calculators, mathematical instruments, pictures, videos, audios and personal handwritings. For instance, given the quadratic equation  $ax^2 + bx + c = 0$ , where the roots are

$$x_1 = \frac{b + \sqrt{b^2 - 4ac}}{2a}$$
 and  $x_2 = \frac{b - \sqrt{b^2 - 4ac}}{2a}$ , because only symbols are used, the interpreted relationships of the

mathematical objects are characterized as symbolic, the signs involving the visualization of the formula are the iconic, and the actions of substituting values for the constants a, b, and c in order to solve for  $x_1$  and  $x_2$  are indexical (Ali & Wilmot, 2016;Benning & Agyei, 2016; Van den Heuvel-Panhuizen et al., 2016).

# Methodology

This research method was mixed methods concurrent and adopted both quantitative and qualitative approach. The instruments for the study were questionnaire/teacher-made tests and interview guide. In the teacher-made tests, two sets of twenty questions were set and distributed to the respondents. The sets of questions were similar and each twenty questions were based on De Saussure and Peirce Semiotic Models. At the same time, twenty semi-structured interview items were set and distributed to five respondents.

# Research Goal

The study therefore, sought to address the following issues:

1. Are there statistically significant differences between the teaching and learning of quadratic equations with De Saussure and Peirce models?

2 What results emerge from comparing the exploratory qualitative data about De Saussure and Peirce models and the results of the quantitative tests of significances?

# The research design

The concurrent mixed methods triangulation design will be used, and it is a design in which one set provides equal supportive roles for another data set (Cohen et al., 2007; Creswell, 2014).



Figure 2: Concurrent Mixed Methods Triangulation Design (Credit: Creswell, 2014)

The concurrent mixed methods triangulation design was used to address the effectiveness of employing semiotic objects in solving for the roots of quadratic equation. The concurrent triangulation mixed methods design is a type of design in which two complementary data sets were collected on the same quadratic equations. In this study, quantitative tests (quantitative instruments) were used to test the theory of semiosis that predicts how age, teaching experience and gender of a school (independent variables) influenced significantly positively the student teachers' effective use of De Saussure and Peirce semiotic models to solve for the roots of quadratic equations (dependent variables) in level 300 post diploma students of the Department of Basic Education in the University of Education, Winneba.

Concurrent with this quantitative data collection was a qualitative interview guide of open questions to explore effectiveness of using the two semiotic models for the same group of student teachers in the same site. Since the study was concerned as to whether Peirce model is more extensive, adequate, excellent and deeper, and whether it made no difference with De Saussure model, all the 45 student teachers were explored with De Saussure model (i.e. control) and with the Peirce model (i.e. experimental) with a Likert scale, and the differences between the two semiotic models were compared. The reasons for collecting both the quantitative and qualitative data were to bring together the strengths of both forms of research, compare, validate and corroborate the results (Cohen et al., 2007; Creswell, 2014).

# Reliability and validity of instruments

The reliability of the quantitative instruments was measured by Cronbach's alpha coefficient at 0.78. Because the instruments were set on two models, the split-half reliability was used. Validity of the quantitative was measured by content and construct. The content validity was satisfied by ensuring that the items in the instruments emanated from the course outline. Construct validity was met by ensuring that the items contain the right terminology and concepts from quadratic expressions and equations. On the qualitative instruments, validity and reliability were satisfied by involving students who really studied quadratic equations and contacting lecturers and other specialists in mathematics education in the university (Creswell, 2014).

# Sample and Data Collection

Out of a total population of 75 enrolled to study after the regular vacation (sandwich) period, the study sampled 45, 49% female and 51% male student teachers. In terms of their teaching experiences in quadratic equations, 33% had three years, 26% four years, 20% five years, and 21% over five years' teaching experience in the public junior high school teaching in Ghana. In terms of the sex of school, 33% taught in male schools only, 25% females only, and 24% mixed. The sampling procedure was purposive and studied only student teachers who taught mathematics in the Basic schools in Ghana.

# Analyzing of Data

On quantitative analysis, since the data contained three categorical *independent variables* (such as rank, and position, teaching experience) and *continuous dependent variables* (such as Pierre and De Saussaure models, objects of Peirre and De Saussaure models, and diachronicity, trichronicity, categorization and quadratic equations), the researchers ensured that the data satisfied normality and independent assumptions t-test and ANOVA techniques. Thereafter, upon being satisfied that the measurements are fitting to all the assumptions especially normality of t-test and ANOVA, the researchers used paired samples t-test to compare the student teachers' effectiveness of De Saussure and Peirce models, and explored multiple ANOVA to compare the effectiveness of all semiotic objects within De Saussure and Peirce models, where a pretest was used to identify and remove possible covariates and established the learning gaps in the two models. On the qualitative analysis, the ordinary (expected) themes were employed to report the effectiveness and satisfied the reliability of the qualitative analysis with anonymity, confidentiality and privacy of respondents (Lai, 2013).

#### Results

In responding to the research question, *are there statistically significant differences between the teaching and learning of quadratic equations with De Saussure and Peirce models*, the results in Table 1 and Table 2 adequately addressed the question.

	Paired	Differences					-	-	-
Models		Std.	Std. Error	95% Interval	Confidence	t	df	Sig. (2- tailed)	Effect size
	Mean	Deviation	Mean	Lower	Upper				
Peirce –De Saussure	0.556	0.624	0.093	0.368	0.743	5.976	44	0.000	0.891

Table 1: Paired Samples T-Test between Peirce Objects and De Saussure Models

The paired-samples t-test or repeated measures was used to compare the effectiveness of Peirce and De Saussure models of student teachers learning outcomes in quadratic equations. There was a statistically significant improvement from De Saussure (M=1.44, SD=0.624) to Peirce [M=1.78, SD=1.204, t(45)=5.976, p<0.000] with an effect size of 89.1%. This effect size supports that the chance that for a randomly selected pair of student-teachers the effectiveness of Peirce was higher than the effectiveness of De Saussure model is 89.1%.

Objects	Experience	Ν	Mean	Std. Deviation	Minimum	Maximum
	Three Years	18	1.78	0.647	1	3
Peirce Objects	Four Years	14	2.43	1.158	1	4
	Five Years	11	1.73	0.905	1	4
	<b>Over Five Years</b>	2	2	0	2	2
	Total	45	1.98	0.917	1	4
	Three Years	18	2	0	2	2
De Saussaure Objects	Four Years	14	2	0	2	2
	Five Years	11	2	0	2	2
	Over Five Years	2	2	0	2	2
	Total	45	2	0	2	2

Table 2: Descriptive Statistics Peirce Objects and De Saussure Objects

Table 2 shows the descriptive we used to peruse the characteristics of teachers using both Peirce and de Saussaure objects in teaching quadratic. It is instructive to note that almost all the 45 student-teachers who used De Saussaure objects had a minimum and maximum time of two years' of teaching experience. Again, the mean scores were 2.00 for all years' of teaching experience with absolutely no variations. However, this story was completely different when it

came to using Peirce objects. Even though about 96% (43) of the student-teachers had more than two years' of teaching experience, the mean scores were mostly lower than those of De Sausaure, coupled with high standard deviations around these means. This situation therefore requires comprehensive and detailed statistical analysis.

Items		Sum of Squares	df	Mean Square	F	Sig.	Effect Size
General Use of Semiotic	Between Groups	38.672	12	3.223	6.223	0.000	0.700
Objects in Quadratic	Within Groups	16.572	32	0.518			
Equations	Total	55.244	44				
Use of De Saussure Objects	Between Groups	8.044	12	0.670	1.537	0.162	0.366
in Quadratic Equations	Within Groups	13.956	32	0.436			
	Total	22.000	44				
Use of Peirce objects in	Between Groups	38.978	12	3.248	4.191	0.001	0.611
Quadratic Equations	Within Groups	24.800	32	0.775			
	Total	63.778	44				

Table 3: Multiple ANOVA Comparisons of Peirce Objects and De Saussure Models

Table 3 shows the multiple ANOVA tests on the effectiveness of Peirce and De Saussure models in the teaching and learning of quadratic equations. Under the general use, teaching and learning with both Peirce and De Saussure models had generally been effective (F=6.223, p=0.000), with 70% of the variance due to teaching experience. De Saussure objects, teaching and learning of quadratic equations had not been effective (F=1.537, p=1.620), with 36.6% of the variance due to teaching experience but under Peirce objects, teaching and learning had been comprehensively effective (F=4.191, p<0.0001), with 61.1% of the variance due to teaching experience. Thus, while there were significant improvements for the teaching and learning of quadratic equations with Peirce model, there was no improvement in employing the De Saussure model.

	Table 4: Qualitative	Analysis of the	e Effectiveness	of Semiotic O	bjects
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Semiotic Objects	Models	Most	Very	Average	Below	average
		effective	effective	effective	effective	
Diachronicity	De Saussure	40%	45%	50%	55%	
	Peirce	60%	55%	50%	45%	
Triachronicity	De Saussure	25%	25%	45%%	65%	
	Peirce	75%	75%	55%	35%	
Categorization	De Saussure	20%	20%	55%	70%	
	Peirce	80%	80%	45%	30%	
Quadratic Equations	De Saussure	15%5	25%	55%	80%	
	Peirce	85%	75%	45%	20%	

We discovered on Table 4 that under diachronicity, De Saussure model was 55% most effective and 40% below average effective while Peirce model was 60% most effective and 45% below effective. Under triachronicity, De Saussure model was 25% most effective and 65% below average effective while Peirce model achieved 75% most effective and 35% least effective. Under categorization, De Saussure model attained 0nly 20% most effective and as high as 70% least effective while Peirce model achieved 80% most effective and as small as 30% as least effective. Under quadratic equations, De Saussure model scored just 15% as most effective and high score of 80% least effective while Peirce model scored as high as 85% most effective and as low as 20% least effective.

# Discussions

With regards to whether there were statistically significant differences between the teaching and learning of quadratic equations with De Saussure and Peirce models, the results of the paired samples t-test between Peirce objects and De Saussure models on Table 1 show that there was a statistically significant difference [M=1.78, SD=1.204, t(45)=5.976, p<0.000], with an effect size of 89.1%. This is quite higher than the recommended power of 0.80 (Fiedler et al., 2012).And in advancing to make multiple ANOVA comparisons of Peirce objects and De Saussure objects, the results on Table 3 shows that De Saussure objects (F=4.191, p<0.0001), with a larger size of 61.1%. We could therefore conclude that there are statistically significant differences between the teaching and learning of quadratic equations using De Saussure, and using Peirce models.

The most obvious difference between the Saussurean and Peircean model is being triadic rather than dyadic, for which Peirce's model of the sign features this third *object* (or referent). Again, the difference also arose because the *signified* Saussaure model is not an external referent but an abstract mental representation, for which reason many student-teachers struggled to conceive and apply in quadratic equations (Thornbury, 2011). However, Peirce's *objects* are not confined to physical things and like Saussure's, they always come with already embedded abstract concepts. Moreover, the Peirce's model explicitly allocates places for resources or teaching-learning materials like calculators and computers and for reality outside the sign system which Saussure's model did not directly feature. Furthermore, Peirce's model enabled the student-teachers to operate as a more general model of teaching and learning rather than dyadic model of classroom two-way interaction (Thornbury, 2011).

The difference can also be traced to theory. For Saussure, *signifier* and *signified* are inseparable, one does not exist without the other or one always implicates the other. They are each the other's condition of possibility. For Peirce, however, there is a third element that is necessary for signification to occur and that third force is the *interpretant*, or the understanding of the relation between signifier and signified. This interpretant links the signifier with the signified focuses more attention on the relationship between the signifier and signified (Bartolini Bussi & Mariotti, 2008). This becomes a major turning point in ensuring that student-teachers transformed cultural artefacts in the quadratic expressions into quadratic equations and solve problems thereafter (Davis & Chaiklin, 2015).

We also tallied these quantitative analyses with open-ended qualitative analyses to compare and corroborate the outcomes. In responding to the research question on *what results emerge from comparing the exploratory qualitative data about De Saussure and Peirce models*, descriptive accounts which involve diachronicity, triachronicity, categorization and quadratic equations under the semiotic models were analysed. The analysis explored the data and generated descriptions that are conceptually pure, meaningful and illuminating around the thematic areas (e.g. De Saussure's groupings of synchronicity and diachronicity into signifier and signified, Peirce's groupings of triadic into representamens, interpretants and interpretents, Peirce categorization of signs into icon, index and symbol and types of factorizations of quadratic equations) (Lanir, 2019; Saenz-Ludlow & Kadunz, 2016).

The following summarized statements are clear indications that the student-teachers really adjudged Peirce model as the most effective under:

- 1. Peirce model is broader, comprehensive and universal.
- 2. Peirce model has been broken down into smaller pieces, emphasizing all parts of signs and embossing all mathematical concepts.
- 3. Peirce model is integrable, innovative and radical in transforming mathematics teaching and learning, including quadratic equations.
- 4. Peirce model is simpler, easier, digital and ICT-compliant (Presmeg et al., 2016).

# Conclusion

The tests of significances of the quantitative data indicate that semiotic objects are very effective tools for facilitating construction of knowledge about quadratic factorizations and solving for the roots. The design and exploration of the two models within semiosis enabled student teachers to gain deep insights into the factorizations of quadratic equations to construct knowledge about the relationships between the techniques of factorizing quadratic equations. For instance, during the processes of solving for the roots of the quadratic equation, the student teachers first explored the relationship between the common factor, grouping, regrouping, difference of two squares, perfect squares, completing squares and quadratic formula.

The student teachers then constructed their solutions by consolidating the semiotic mediations into the semiotic objects (representamens, interpretants and interpretents), outlining the variables and constants into calculators, programming the quadratic functions, and recording the roots of the quadratic equations. These rich and well-resourced algorithms in the diachronicity (e.g., the signifier and signified), triachronicity (e.g. representamens, interpretants and interpretents), categorizations (e.g. icon, index and symbol), and quadratic factorizations (e.g., commons, groups, regroups, differences and formula) enabled the student teachers to construct effective knowledge, gained deep understanding of semiosis and improved upon their learning outcomes.

# Recommendations

Based on the findings, we made the following recommendations for theory, practice and policy:

Student-teachers should improve upon the cultural outlook of objects. It was really hardly successful without the interpretant. If Peirce's model was not used, it would have been a total failure in achieving success of teaching and learning quadratic equations.

Secondly, student-teachers should constantly practise the tradium. Any lack of experiential teaching and learning would rather the Pierre's model null and void.

Finally, professions of the student-teachers show that Peirce's model was most successful. We therefore recommended the model to policy makers to conclude it into the curricular of all levels of education.

The meaning and usage of semiotic models are dynamic and constantly challenging, and the semiotic objects are continuously being re-examined as they evolve with time. Therefore, based on the findings and feedbacks of this study, we suggest that any research in semiotic models should be redesigned and re-examined in much wider area and larger samples. Concurrently, we also suggest that future semiotic frameworks should be revised and modified on an extended scope and wider body of knowledge in Conic Sections in general.

#### Limitations

The sample size was rather small for this study to be generalized. Because of this, the findings were also too skewed to one perspective. Further studies should expand the scope of semiosis.

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